

Maróthi György Memorial Competition,
October 30 – November 30, 2020
– Rules –

Description

The Mathematical Institute of the University of Debrecen announces a competition in Mathematics for BSc students of the University of Debrecen in their first or second year of studies during the fall semester of the academic year 2020/2021. The competition is individual, registration is not required. The list of problems is published at noon on October 30, 2020, on the web page of the Institute:

www.math.unideb.hu

Available at: [Hallgatóinknak](#) » [Versenyek](#) » [Maróthi György Emlékverseny](#)

Organizers

<i>dr. Boros Zoltán</i>	<i>(Coordinator of Talent Management in the Institute of Mathematics, Department of Analysis)</i>
<i>dr. Bessenyei Mihály</i>	<i>(Competition Secretary, Department of Analysis)</i>
<i>dr. Szilasi Zoltán</i>	<i>(Department of Geometry)</i>
<i>dr. Pongrácz András</i>	<i>(Department of Algebra and Number Theory)</i>
<i>dr. Horváth Gábor</i>	<i>(Morgan Stanley)</i>

Sponsorship

Organizers thank the financial support by the Morgan Stanley Magyarország Elemző Kft.

Formal requirements

Solutions should be submitted in English (or Hungarian), written by hand on blank sheets of papers. Write your name and the (ordinal) number of the problem which is elaborated on that sheet (e.g.: Karol Grande, Problem 5) to the top of the page. Solutions to distinct problems should be elaborated on separate sheets of paper. The photo or scan of the hand written solutions have to be sent by email to Zoltán Boros (zboros@science.unideb.hu) and Mihály Bessenyei (besse@science.unideb.hu). In the text of the mail, please indicate Your name, NEPTUN code, major (mathematics, physics, ... engineering, ... management, information technology, etc.) and the year (first or second at BSc) of studies.

Deadline for submission: November 30, 2020, Monday, 12:00. (noon)

Ethical regulation

Though all problems can be solved using standard college mathematics, you can use any additional sources if it is appropriately cited in your solution. Cooperation of the participants (with each other or with any other person on any platform) is not allowed. If such a cooperation is established, all involved participants will be disqualified.

Every participant will be notified of his/her result.

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– Problems –

Problem 1. *A circle is tangent to the side BC of the triangle ABC (from outside) at the point D , while it is also tangent to the line AB at the point E and to the line AC at a third point. The bisector line of the external angle at the edge C intersects the line DE at the point G . Determine the measure of the angle GAC in terms of the measures of the internal angles of the triangle ABC .*

(Proposed by: Zoltán Szilasi)

Problem 2. *In the isosceles triangle ABC the length of the sides AB and AC are equal. Let O denote the center of the circumscribed circle of the triangle, and let F be the bisection point of the side AB . Let S denote the centroid (center of mass) of the triangle AFC . Prove that the line CF is perpendicular to the line SO .*

(Proposed by: Zoltán Szilasi)

Problem 3. *Suppose that $x \in \mathbb{R}$ such that $x + x^{-1} \in \mathbb{Z}$. Prove $x^n + x^{-n} \in \mathbb{Z}$ for every $n \in \mathbb{Z}$.*

(Proposed by: András Pongrácz)

Problem 4. *Prove the identity*

$$\sum_{k=0}^{n-1} \binom{n+k-1}{k} \frac{1}{2^{n+k}} = \frac{1}{2}.$$

for every $n \in \mathbb{N}$.

(Proposed by: András Pongrácz)

Problem 5. *Is it possible to divide the set of positive rational numbers into two non-empty, disjoint parts such that each part is closed with respect to addition?*

(Proposed by: Mihály Bessenyei)

Problem 6. *Determine the minimum of the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by the formula $f(x) := 4^x + 2^{3-x}$.*

(Proposed by: Zoltán Boros)

Solution to each problem is evaluated up to 5 points.
The order of the problems need not indicate their difficulty.