Bilocal automorphisms *

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Abstract

We prove that every bilocal automorphism of a matrix algebra is either an inner automorphism, or an inner anti-automorphism, or it is of a very special degenerate form. Bijective continuous bilocal automorphisms of a unital standard operator algebra on an infinite-dimensional separable complex Banach space are automorphisms.

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1 Introduction and statement of the main results

In 1990, Kadison, and Larson and Sourour [3, 4] initiated the study of local derivations and local automorphisms. Let $\mathcal{B}$ be an algebra. Then a linear map $\phi : \mathcal{B} \to \mathcal{B}$ is called a local automorphism (a local derivation) if for every $x \in \mathcal{B}$ there exists an automorphism (a derivation) $\phi_x : \mathcal{B} \to \mathcal{B}$ (depending on x) such that $\phi(x) = \phi_x(x)$. The question is, of course, under what conditions every local automorphism (local derivation) must be an automorphism (a derivation). There is a vast literature on local maps. Many references can be found in the book [5].

Let X be a complex Banach space and $\mathcal{B}(X)$ the algebra of all bounded linear operators on X. Larson and Sourour [4] proved that every local derivation of $\mathcal{B}(X)$ is a derivation. Moreover, every surjective local automorphism of $\mathcal{B}(X)$ is an automorphism provided that X is infinite-dimensional. If X is finite-dimensional, $\dim X = n$, then we can identify $\mathcal{B}(X)$ with the algebra $M_n(\mathbb{C})$ of all $n \times n$ complex matrices. Larson and Sourour proved that every local automorphism of $M_n(\mathbb{C})$ is either an automorphism or an anti-automorphism. Note that they did not need the surjectivity assumption in the finite-dimensional case.

A surprising extension of the result of Larson and Sourour on local derivations was given in [11]. Let us first recall that a closed subalgebra $\mathcal{A} \subset \mathcal{B}(X)$ is called standard if $\mathcal{F}(X) \subset \mathcal{A}$, where $\mathcal{F}(X) \subset \mathcal{B}(X)$ is the subset of all finite rank operators. The algebra $\mathcal{A}$ is unital if it contains the identity. Zhu and Xiong defined a linear map $\phi : \mathcal{A} \to \mathcal{B}(X)$ to be a bilocal derivation if for every $T \in \mathcal{A}$ and $x \in X$ there exists a derivation $\phi_{T,x} : \mathcal{A} \to \mathcal{B}(X)$, depending on both T and x, such that $\phi(T)x = \phi_{T,x}(T)x$. Clearly, every local derivation is a bilocal derivation. Zhu and Xiong proved that if $\mathcal{A}$ is a unital standard operator algebra on a complex Banach space X and $\phi : \mathcal{A} \to \mathcal{B}(X)$ is a bilocal derivation, then ϕ must be a derivation.

Recently, Molnár [6] showed that bilocal $*$-automorphisms (the notion should be self-explanatory) on operator algebras are not necessarily $*$-automorphisms. More precisely, if H is an infinite-dimensional separable Hilbert space, then $\phi : \mathcal{B}(H) \to \mathcal{B}(H)$ is a bilocal $*$-automorphism if and only if ϕ is a unital $*$-endomorphism of $\mathcal{B}(H)$. In the finite-dimensional case ϕ is a bilocal $*$-automorphism if and only if ϕ is a Jordan $*$-automorphism. At the end of the paper [6] he asked what happens if the group of $*$-automorphisms of $\mathcal{B}(H)$ is replaced by the larger group of all automorphisms of $\mathcal{B}(H)$.

This research was motivated by the above question. In the finite-dimensional case we do not want to restrict ourselves to matrices over the complex field, and therefore we will introduce the notion of bilocal automorphisms for operator algebras on vector spaces over an arbitrary field.

Let U be a linear space over a field $\mathbb{F}$, $\mathcal{L}(U)$ the algebra of all linear operators on U, and $\mathcal{A} \subset \mathcal{L}(U)$ a subalgebra of $\mathcal{L}(U)$. A linear map $\phi : \mathcal{A} \to \mathcal{A}$ is said to be a bilocal automorphism if for every $T \in \mathcal{A}$ and every $u \in U$ there exists an automorphism $\phi_{T,u}$ of $\mathcal{A}$, depending on T and u, such that

$$\phi(T)u = \phi_{T,u}(T)u.$$

We will be interested in two special cases. In the first case where $\dim U = n < \infty$ and $\mathcal{A} = \mathcal{L}(U)$ we identify $\mathcal{A}$ with $M_n(\mathbb{F})$, the algebra of all $n \times n$ matrices over $\mathbb{F}$. Then U is identified with the set $\mathbb{F}^n$ of all $n \times 1$ matrices over $\mathbb{F}$. For a matrix $T \in M_n(\mathbb{F})$ we denote its transpose by T^{tr}. The second case we want to consider is when $U = X$ is a complex Banach space and $\mathcal{A}$ is a unital standard operator algebra on X.

In order to formulate our first result we need to introduce the notion of full nonsingular subspaces of $M_n(\mathbb{F})$. A linear subspace $\mathcal{V} \subset M_n(\mathbb{F})$ is called a nonsingular subspace if every nonzero element of $\mathcal{V}$ is invertible. It is called a full nonsingular subspace if in addition $\dim \mathcal{V} = n$. Some remarks on the existence of such subspces can be found in the last section of this note.

Theorem 1.1 *Let $\phi : M_n(\mathbb{F}) \to M_n(\mathbb{F})$ be a linear map. Then ϕ is a bilocal automorphism if and only if one of the following holds:*

- *there exists an invertible matrix $A \in M_n(\mathbb{F})$ such that $\phi(T) = ATA^{-1}$, $T \in M_n(\mathbb{F})$, or*
- *there exists an invertible matrix $A \in M_n(\mathbb{F})$ such that $\phi(T) = AT^{tr}A^{-1}$, $T \in M_n(\mathbb{F})$, or*
- *there exist a full nonsingular subspace $\mathcal{V} \subset M_n(\mathbb{F})$, a nonzero $x \in \mathbb{F}^n$, and a linear bijection $\alpha : \mathbb{F}^n \to \mathcal{V}$ satisfying $\alpha(x) = I$ such that $\phi(T) = \alpha(Tx)$, $T \in M_n(\mathbb{F})$, or*
- *there exist a full nonsingular subspace $\mathcal{V} \subset M_n(\mathbb{F})$, a nonzero $x \in \mathbb{F}^n$, and a linear bijection $\alpha : \mathbb{F}^n \to \mathcal{V}$ satisfying $\alpha(x) = I$ such that $\phi(T) = \alpha(T^{tr}x)$, $T \in M_n(\mathbb{F})$.*

In the infinite-dimensional case we have the following result.

Theorem 1.2 *Let X be an infinite-dimensional separable complex Banach space and $\mathcal{A}$ a unital standard operator algebra on X. Assume that a linear map $\phi : \mathcal{A} \to \mathcal{A}$ is continuous and bijective. Then ϕ is a bilocal automorphism if and only if it is an automorphism.*

As we will show in the last section the assumptions of bijectivity and separability are indispensable in the above statement. We conjecture that the same conclusion holds without the continuity assumption. Some remarks on this conjecture and a related open problem will be presented at the end of this note.

2 Proofs

Let U be a linear space over a field $\mathbb{F}$ and $T : U \to U$ a linear operator. We denote by $\sigma_p(T)$ the set of all eigenvalues of T (in the case when U is a complex Banach space and T is a bounded operator, this set is called the point spectrum of T).

We will call a subalgebra $\mathcal{A} \subset \mathcal{L}(U)$ a regular operator algebra on U if the following conditions are fulfilled:

- $I \in \mathcal{A}$,
- every automorphism ϕ of $\mathcal{A}$ is spatial, that is, there exists an invertible $A \in \mathcal{L}(U)$ such that $\phi(T) = ATA^{-1}$, $T \in \mathcal{A}$, and
- for every pair of linearly independent vectors $u, v \in U$ and every pair of linearly independent vectors $x, y \in U$ there exists an invertible $A \in \mathcal{A}$ such that $Ax = u$ and $Ay = v$.

The following observation is crucial in the proofs of our main results.

Proposition 2.1 *Let $\mathcal{A}$ be a regular operator algebra on U and $\phi : \mathcal{A} \to \mathcal{A}$ a linear map. Then ϕ is a bilocal automorphism if and only if ϕ is unital and for every $T \in \mathcal{A}$ we have $\sigma_p(\phi(T)) \subset \sigma_p(T)$.*

Proof. Assume first that ϕ is a bilocal automorphism. Then clearly, ϕ is unital. Suppose that $\lambda \in \mathbb{F}$ is an eigenvalue of $\phi(T)$, that is, we have $\phi(T)u = \lambda u$ for some nonzero $u \in U$. We know that

$$\phi(T)u = ATA^{-1}u$$

for some invertible $A \in \mathcal{L}(U)$. It follows that

$$T(A^{-1}u) = A^{-1}\phi(T)u = A^{-1}(\lambda u) = \lambda(A^{-1}u),$$

and thus, λ is an eigenvalue of T, as desired.

To prove the other direction, assume that ϕ is unital and that $\sigma_p(\phi(T)) \subset \sigma_p(T)$ for every $T \in \mathcal{A}$. We need to prove that ϕ is a bilocal automorphism, that is, for every $T \in \mathcal{A}$ and every $u \in U$ we need to find an automorphism $\phi_{T,u}$ of $\mathcal{A}$ such that $\phi(T)u = \phi_{T,u}(T)u$. We will prove even more, namely that $\phi_{T,u}$ can be chosen to be inner. To see this we have to show that for every $T \in \mathcal{A}$ and every $u \in U$ there exists an invertible operator $A \in \mathcal{A}$ such that $\phi(T)u = ATA^{-1}u$. Obviously, this is trivial when $T = 0$ or $u = 0$. So, assume they are both nonzero. Set $v = \phi(T)u$. We will distinguish two possibilities.

Let us first assume that u and v are linearly dependent. Since $u \neq 0$, we have $v = \lambda u$ for some scalar λ. Hence, $\phi(T)u = \lambda u$, and therefore, $\lambda \in \sigma_p(\phi(T)) \subset$

$\sigma_p(T)$. Consequently, there exists a nonzero $z \in U$ such that $Tz = \lambda z$. We can find an invertible operator $A \in \mathcal{A}$ such that $Az = u$. Then

$$ATA^{-1}u = ATz = \lambda Az = \lambda u = v = \phi(T)u.$$

The proof in the first case is completed.

It remains to consider the case when u and v are linearly independent. In particular, $\phi(T) \notin \mathbb{F}I$, and because ϕ is unital, $T \notin \mathbb{F}I$. It follows that we can find linearly independent vectors $x, y \in U$ such that $Tx = y$. Moreover, there exists an invertible $A \in \mathcal{A}$ satisfying $u = Ax$ and $v = Ay$. Then again

$$ATA^{-1}u = ATx = Ay = v = \phi(T)u.$$

□

Once we have the above statement, the main result in the finite-dimensional case follows rather easily from the recent characterization of linear preservers of invertibility on matrix algebras due to de Seguins Pazzis [8].

Proof of Theorem 1.1. Clearly, if ϕ is an automorphism, then it is a bilocal automorphism. It is well-known (see for example [10]) that every matrix is similar to its transpose, and therefore, every anti-automorphism is a bilocal automorphism as well. Assume next that there exist a full nonsingular subspace $\mathcal{V} \subset M_n(\mathbb{F})$, a nonzero vector $x \in \mathbb{F}^n$, and a linear bijection $\alpha : \mathbb{F}^n \to \mathcal{V}$ satisfying $\alpha(x) = I$ such that $\phi(T) = \alpha(Tx)$. Then $\phi(I) = I$. Moreover, if T is invertible, then $Tx \neq 0$, and consequently, $\phi(T) = \alpha(Tx)$ is invertible. In order to prove that ϕ is a bilocal automorphism we first observe that $M_n(\mathbb{F})$ is a regular operator algebra and then all we need to show is that $\sigma_p(\phi(T)) \subset \sigma_p(T)$ for every $T \in M_n(\mathbb{F})$. Assume that $T \in M_n(\mathbb{F})$ and let λ be a scalar such that $\lambda \notin \sigma_p(T)$. Then $\lambda I - T$ is invertible, and hence $\lambda I - \phi(T)$ is invertible, or equivalently, $\lambda \notin \sigma_p(\phi(T))$, as desired. In the same way we verify that ϕ of the fourth form given in the conclusion of the theorem is a bilocal automorphism.

Now, assume that ϕ is a bilocal automorphism. Then it is unital. Moreover, it preserves invertibility. Indeed, all we need to show is that for every invertible $T \in M_n(\mathbb{F})$, the operator $\phi(T)$ has trivial kernel. This follows directly from the definition of bilocal automorphisms. Hence, we can apply the main theorem from [8] which characterizes linear maps on $M_n(\mathbb{F})$ preserving invertibility. The desired conclusion follows directly from [8, Theorem 2] and the fact that ϕ is unital.

□

We now turn to the proof of our second main result.

Proof of Theorem 1.2. By the result of Chernoff [1], every automorphism of $\mathcal{A}$ is spatial. Assume that $x, y \in X$ are linearly independent and that also

$u, v \in X$ are linearly independent. Then X can be writen as a direct sum $X = \text{span}\{x, y, u, v\} \oplus Z$ for some closed subspace Z. Clearly, there exists an invertible linear operator $A_1 : \text{span}\{x, y, u, v\} \to \text{span}\{x, y, u, v\}$ satisfying $A_1 x = u$ and $A_1 y = v$. Define a linear operator $A : X \to X$ by $Az = A_1 z$ if $z \in \text{span}\{x, y, u, v\}$ and $Az = z$ whenever $z \in Z$. Obviously, $Ax = u$, $Ay = v$, A is a bounded invertible linear operator and $A \in \mathbb{C}I + \mathcal{F}(X) \subset \mathcal{A}$. Hence, the algebra $\mathcal{A}$ is a regular operator algebra.

By the open mapping theorem the inverse of the bilocal automorphism ϕ is bounded. We denote it by ψ. We know that ϕ is unital and then the same must be true for ψ. It is our aim now to show that for every $T \in \mathcal{A}$ we have

$$\sigma_{ap}(T) \subset \sigma_{ap}(\psi(T)).$$

Here, $\sigma_{ap}(T)$ denotes the approximate point spectrum of T, that is, the set of all complex numbers λ such that $T - \lambda$ is not bounded below.

So, assume that $\lambda \in \sigma_{ap}(T)$. Then we can find a sequence (x_n) of vectors of norm one in X such that $\|(T - \lambda)x_n\| \to 0$. Set $(T - \lambda)x_n = y_n$. By Hahn-Banach theorem we can find a sequence (f_n) of norm one functionals in the dual space X' such that $f_n(x_n) = 1$, $n = 1, 2, \ldots$ For each positive integer n we introduce an operator $S_n = T - \lambda - y_n \otimes f_n$. Here, for any $z \in X$ and $g \in X'$ the operator $z \otimes g : X \to X$ is defined by $(z \otimes g)w = g(w)z$, $w \in X$. Because f_n is a functional of norm one we have $\|y_n \otimes f_n\| \le \|y_n\|$, and consequently, the sequence (S_n) of operators in $\mathcal{A}$ tends to $T - \lambda$.

We know that $\sigma_p(\phi(T)) \subset \sigma_p(T)$ for every $T \in \mathcal{A}$. It follows that $\sigma_p(T) \subset \sigma_p(\psi(T))$ for every $T \in \mathcal{A}$. This together with $S_n x_n = 0$ yields that $0 \in \sigma_p(\psi(S_n))$ for every positive integer n. By the continuity of the map ψ, the sequence $(\psi(S_n))$ converges to $\psi(T) - \lambda$. We claim that $0 \in \sigma_{ap}(\psi(T) - \lambda)$. Indeed, if this was not true, then there would exist a positive constant c such that $\|(\psi(T) - \lambda)w\| \ge c$ for every $w \in X$ of norm one. In particular, this would be true for each w belonging to the kernel of $\psi(S_n)$. As this kernel is nontrivial for every positive integer n, we would have that the distance between $\psi(T) - \lambda$ and $\psi(S_n)$ is at least c, a contradiction. Hence, $0 \in \sigma_{ap}(\psi(T) - \lambda)$, and consequently, $\lambda \in \sigma_{ap}(\psi(T))$, as desired.

We have proved that $\sigma_{ap}(T) \subset \sigma_{ap}(\psi(T))$ for every $T \in \mathcal{A}$. Equivalently, $\sigma_{ap}(\phi(T)) \subset \sigma_{ap}(T)$ for every $T \in \mathcal{A}$. By [2, Theorem 4.4], ϕ is either a spatial automorphism, or a spatial anti-automorphism. In order to complete the proof we need to show that the second case cannot occur.

Assume on the contrary, that we have the second possibility, that is, there exists an invertible bounded linear map $A : X' \to X$ such that $\phi(T) = AT'A^{-1}$ for every $T \in \mathcal{A}$. We will show that there exists $T \in \mathcal{A}$ such that T is injective but T' is not injective. Assume for a moment that we have already found such an operator T. Then we can find a nonzero $x \in X$ such that $AT'A^{-1}x = \phi(T)x = 0$. On the other hand, since ϕ is a bilocal automorphism, there exists a bounded invertible operator B such that $\phi(T)x = BTB^{-1}x \ne 0$, a contradiction.

Thus, we will complete the proof by finding $T \in \mathcal{A}$ being injective and having a noninjective adjoint. Since X is separable, we can apply the result of Ovsepian and Pełczyński [7] on the existence of a fundamental and total biorthogonal sequence $(x_n, f_n) \subset X \times X'$ that satisfies the condition $\sup \|x_n\| \|f_n\| = M < \infty$. Define

$$T = \sum_{n=1}^{\infty} 2^{-n} x_{n+1} \otimes f_n$$

and observe that since $\sum_{n=1}^{N} 2^{-n} x_{n+1} \otimes f_n \in \mathcal{A}$ for every positive integer N and $\mathcal{A}$ is closed, the operator T belongs to $\mathcal{A}$. If $Tx = 0$ for some $x \in X$, then $\sum_{n=1}^{\infty} 2^{-n} f_n(x) x_{n+1} = 0$, and consequently, $f_n(x) = 0$ for every positive integer n which yields that $x = 0$. On the other hand,

$$T' = \sum_{n=1}^{\infty} 2^{-n} f_n \otimes \kappa x_{n+1},$$

where κ is the canonical embedding of X into X'', and then obviously, $T' f_1 = 0$.

□

3 Final remarks

Let n be an integer, $n \geq 2$. If $\mathbb{F}$ is algebraically closed, then there are no full nonsingular subspaces of $M_n(\mathbb{F})$. On the other hand, if $\mathbb{F}$ is a finite field of cardinality p^s, p prime, then there exist full nonsingular subspaces of $M_n(\mathbb{F})$ for every $n \geq 2$. And finally, $M_n(\mathbb{R})$ contains a full nonsingular subspace if and only if $n \in \{2, 4, 8\}$. All these are well-known facts. For a more detailed explanation we refer to [8, 9].

We complete this note by making some remarks on the indispensability of the bijectivity, separability, and continuity assumptions in Theorem 1.2. We first show that without the bijectivity assumption the behaviour of bilocal automorphisms may be quite wild. Indeed, let H be an infinite-dimensional Hilbert space. Then H can be identified with $H \oplus H$, and consequently, the elements of $\mathcal{B}(H)$ can be represented as 2×2 matrices with entries in $\mathcal{B}(H)$. Let $\varphi : \mathcal{B}(H) \to \mathcal{B}(H)$ be any linear map satisfying $\varphi(I) = 0$. We define $\phi : \mathcal{B}(H) \to \mathcal{B}(H)$ in the following way. We first identify the target space with the algebra of all 2×2 operator matrices and then set

$$\phi(T) = \begin{bmatrix} T & \varphi(T) \\ 0 & T \end{bmatrix}, \quad T \in \mathcal{B}(H).$$

The map ϕ is clearly unital. It is straightforward to verify that $\sigma_p(\phi(T)) \subset \sigma_p(T)$ for every $T \in \mathcal{B}(H)$. By Proposition 2.1, the map ϕ is a bilocal automorphism. However, φ behaves on a complemented subspace of the one-dimensional subspace of scalar operators arbitrarily.

In order to show that Theorem 1.2 does not hold true without the separability assumption, take a nonseparable Hilbert space H with the orthonormal basis $\{e_\alpha : \alpha \in \Delta\}$ and define a conjugate-linear bijective involution $J : H \to H$ by

$$J\left(\sum_{\alpha\in\Delta} \lambda_\alpha e_\alpha\right) = \sum_{\alpha\in\Delta} \overline{\lambda_\alpha} e_\alpha$$

for every $\sum_{\alpha\in\Delta} \lambda_\alpha e_\alpha \in H$. Denote by $\mathcal{K}(H)$ the ideal of all compact operators. For an operator $T \in \mathcal{B}(H)$ we denote by $\sigma(T)$ the spectrum of T. The map $T \mapsto JT^*J$, $T \in \mathcal{B}(H)$, is an anti-automorphism of $\mathcal{B}(H)$, and therefore, $\sigma(T) = \sigma(JT^*J)$ for every $T \in \mathcal{B}(H)$. Since H is nonseparable, we have $\sigma(T) = \sigma_p(T)$ for every $T \in \mathcal{K}(H)$. Indeed, all nonzero members of the spectrum of a compact operator are eigenvalues, and because H is nonseparable, 0 belongs to the point spectrum of each compact operator. Hence,

$$\sigma_p(JT^*J) = \sigma_p(T)$$

for every $T \in \mathcal{K}(H)$, and consequently, $\sigma_p(JT^*J) = \sigma_p(T)$ for every $T \in \mathcal{A} = \mathbb{C}I \oplus \mathcal{K}(H)$. Thus, the map $\phi : \mathcal{A} \to \mathcal{A}$ defined by $\phi(T) = JT^*J$, $T \in \mathcal{A}$, is a bilocal automorphism. However, it is not an automorphism (it is an anti-automorphism).

We believe that Theorem 1.2 holds true without the continuity assumption. It is clear from our proof that one way of proving such a statement would be to show that bijective linear maps on unital standard operator algebras compressing the point spectrum are necessarily automorphisms. This problem is of independent interest. Bijective linear maps on standard operator algebras compressing various parts of spectrum have been studied a lot (see [2] and the references therein). But as far as we know this kind of results have been obtained only for spectral sets having the property that they are nonempty for every operator T. Of course, there are a lot of operators with empty point spectrum. The other possibility would be to employ the automatic continuity techniques and prove directly that bilocal automorphisms are bounded. If one can prove such an automatic continuity result, then one would get the structural result for point spectrum compressing maps as a corollary (see Proposition 2.1).

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