

## Graph Theory and Applications (2017/18, autumn semester)

Detailed course outline:

- *Basic definitions and theorems.*
- *Menger's theorems, multiple connectivity:* vertex, internally vertex and edge disjoint paths, covering vertices and edges, theorems of Menger.  $k$ -edge and  $k$ -vertex connected graphs, vertex and edge connectivity, theorems of Menger, Dirac's theorem. Equivalents of 2-edge and 2-vertex connectedness. Theorems about edge disjoint spanning trees.
- *Chromatic number, perfect graphs, Mycielski graph, chromatic polynomial, list chromatic number, chromatic index, list chromatic index, total chromatic number:* proper vertex colouring, chromatic number, bounds, greedy colouring, theorem of Brooks. Mycielski graph, properties. Perfect graphs, interval graphs. Chromatic polynomial, properties. List chromatic number. Proper edge colouring, chromatic index, properties. List chromatic index. Total chromatic number.
- *Independency and covering, perfect matchings:* independent vertex and edge set, covering vertex and edge set. Independence, matching, vertex cover, edge cover number. Theorems of Gallai and König. Marriage problem, Hall's theorem. Perfect matchings. Tutte's and Petersen's theorems.
- *Augmenting paths, Hungarian maximum matching algorithm:* alternating path, augmenting path. Theorem of Berge. Hungarian maximum matching algorithm.
- *Dominating sets, planar graphs, crossing number:* dominating set, domination number, properties. Some properties of planar graphs, theorem of Fáry and Wagner. Crossing number, bounds.
- *Extremal graph theory:* theorem of Mantel, Turán graph, theorem of Turán.
- *Ramsey numbers.*
- *Friendship theorem:* friendship theorem, generalization. Strongly regular graphs.
- *Directed graphs, tournaments:* basic definitions about directed graphs. Theorem of Gallai and Roy. Theorems of Robins and Nash-Williams. Directed acyclic graphs, topological ordering. Theorem of Stanley. Tournaments. Theorem of Landau. Theorems of Rédei, Szele and Camion.
- *Classes of decision problems:* polynomially solvable, polynomially verifiable, NP-hard problems.

*Examination:* the prerequisite of the exam is passing the practical course.

The type of the examination is written exam. The test will consist of two parts:

1. Direct questions: asking directly for certain definitions, discussed problems, stating and proving theorems.
2. Indirect question: explaining everything about a certain topic in details.