COMBINATORIAL DIOPHANTINE EQUATIONS

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1. Introduction

Many diophantine equations possess combinatorial background. Some special cases have been intensively investigated by several authors (see Table 1). These problems lead to equations of the type

$$f(x) = g(y) \quad \text{in integers} \quad x, y \ ,$$

where f and g are polynomials with rational coefficients of degree three and two, respectively. The purpose of this note is to solve the unsolved equations from Table 1 by using the program package SIMATH [S]. The algorithm is based upon a theorem obtained by Gebel, Pethő and Zimmer [GPZ] and Stroeker and Tzanakis [ST], independently. Our results are summarized in Table 2 through Table 13. We remark that the function `faintp` of SIMATH gives all the integer points on the corresponding elliptic curves within a reasonable amount of CPU-time. As an example, in the case of equation (15) we follow the arguments of [GPZ] directly. In fact, equations (6) and (24) of Table 1 can be treated in an elementary way, and we will deal with these equations separately.

Let $P_k(X)$ denote the polynomial $X(X+1)\dots(X+k-1)$, and set $S_k(X) = 1^k + 2^k + \dots + X^k$. For general results on the equality of these combinatorial numbers we refer to [BP1] and [BP2], respectively.

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No.	Equation	Transformed equation	Reference
1	$P_3(x) = P_2(y)$	$s^3 - 4s + 2 = 2t^2$	Mordell [M]
2	$P_3(x) = P_4(y)$	$s^3 - s + 1 = t^2$	Boyd and Kiselevsky [BK]
3	$P_3(x) = \binom{y}{2}$	$s^3 - 4s + 1 = t^2$	Tzanakis and de Weger [TW]
4	$P_3(x) = \binom{y}{4}$	$s^3 - 36s + 9 = t^2$	Pintér and de Weger [PW]
5	$P_6(x) = P_2(y)$	$s^3 - 10s^2 - 4s + 42 = 2t^2$	MacLeod and Barrodale [MB]
6	$P_6(x) = P_4(y)$	$s^3 - 5s^2 - s + 6 = t^2$	
7	$P_6(x) = \binom{y}{2}$	$s^3 - 10s^2 - 4s + 41 = t^2$	
8	$P_6(x) = \binom{y}{4}$	$s^3 - 30s^2 - 36s + 1089 = t^2$	
9	$\binom{x}{3} = P_2(y)$	$s^3 - 4s + 12 = 3t^2$	
10	$\binom{x}{3} = P_4(y)$	$s^3 - s + 6 = 6t^2$	
11	$\binom{x}{3} = \binom{y}{2}$	$s^3 - 4s + 6 = 6t^2$	Avanesov [A1]
12	$\binom{x}{3} = \binom{y}{4}$	$s^3 - 4s + 2 = 2t^2$	de Weger [dW2]
13	$\binom{x}{6} = P_2(y)$	$s^3 - 5s^2 - s + 185 = 5t^2$	
14	$\binom{x}{6} = P_4(y)$	$s^3 - 5s^2 - s + 725 = 5t^2$	
15	$\binom{x}{6} = \binom{y}{2}$	$s^3 - 5s^2 - s + 95 = 10t^2$	
16	$\binom{x}{6} = \binom{y}{4}$	$s^3 - 5s^2 - s + 35 = 35t^2$	
17	$S_2(x) = P_2(y)$	$s^3 - s + 6 = 6t^2$	
18	$S_2(x) = P_4(y)$	$s^3 - s + 24 = 6t^2$	
19	$S_2(x) = \binom{y}{2}$	$s^3 - s + 3 = 3t^2$	Avanesov [A2]; Uchiyama [U]
20	$S_2(x) = \binom{y}{4}$	$s^3 - s + 1 = t^2$	Boyd and Kiselevsky [BK]
21	$S_5(x) = P_2(y)$	$s^3 - s^2 + 12 = 3t^2$	
22	$S_5(x) = P_4(y)$	$s^3 - s^2 + 48 = 3t^2$	
23	$S_5(x) = \binom{y}{2}$	$s^3 - s^2 + 6 = 6t^2$	
24	$S_5(x) = \binom{y}{4}$	$s^3 - s^2 + 2 = 2t^2$	

Table 1

Theorem 1. *a) All solutions of equation (6) are* $(x, y) = (2, 7),\ (-7, 7),\ (2, -10),\ (-7, -10)$ *and* (x^*, y^*)*, where* $x^* \in \{-5, -4, -3, -2, -1, 0\}$ *and* $y^* \in \{-3, -2, -1, 0\}$.
b) The only solution of equation (24) with $x \geq 1$, $y \geq 0$ *is* $(x, y) = (1, 4)$.

Theorem 2. *All solutions of the unsolved equations of Table 1 are just those which are summarized in the following Table 2 through Table 14.*

At this stage we note that the equation (20) leads to $s^3 - s + 1 = y^2$, which was resolved by Boyd and Kisilevsky [BK] so it is easy to check that there is a unique solution $(x, y) = (2, 5)$ to (20).

2. Proofs and Tables

Proof of Theorem 1. a) Let $f(x) = (x-2)(x-1)x(x+1)(x+2)(x+3)$. Now the equation $f(x) = P_4(y)$ is equivalent to $g(x) := 256x^6 + 768x^5 - 1280x^4 - 3840x^3 + 1024x^2 + 3072x + 256 = a^2$, where $a = 16y^2 + 48y$. Suppose now that $x > 0$ and put $h(x) = 16x^3 + 24x^2 - 58x - 33$. It is easy to verify that for $x \geq 24$ we have $(h(x) - 1)^2 < g(x) < h(x)^2$, which is impossible. Checking the remaining cases, we obtain just the solutions stated in the first part of Theorem 1.

b) The proof of this part is similar. As $S_5(x) = (2x^6 + 6x^5 + 5x^4 - x^2)/12$, the equation $S_5(x) = \binom{y}{4}$ is equivalent to $g(x) := 64x^6 + 192x^5 + 160x^4 - 32x^2 + 16 = a^2$, where $a = 4y^2 - 12y + 4$. We assume again that $x > 0$ and put $h(x) = 8x^3 + 12x^2 + x - 1$. Now one can verify easily that if $x \geq 2$, then $(h(x) - 1)^2 < g(x) < h(x)^2$, which is impossible. Hence the second part of Theorem 2 follows. □

In the followings we give some tables containing the solutions and some other parameters of the equations of Table 1. We give the number of the corresponding equation and the equation itself. Moreover, we include the elliptic curve corresponding to the equation under consideration, and the transformation used to obtain this elliptic curve. At last, we give all those points on this elliptic curve for which there correspondes a solution of the initial equation. For the shake of completeness, after the tables we give all those points on the elliptic curves from which we do not obtain solutions of the initial equations ('other points').

As we mentioned above, to solve the equations of Table 1, we used the program package SIMATH. However, we will give a detailed description of the method used to obtain the solutions only in case of equation (15), just as an example. All the other equations can be treated similarly.

(7)	$P_6(x) = \binom{y}{2}$
Transformation:	$u = 18x^2 + 90x + 60$ $v = 54y - 27$
Elliptic curve:	E_7: $u^3 - 3024u - 33831 = v^2$
Integer points on E_7: $(u, \pm v) =$	Corresponding integer points on (7): $(x, y) =$
(-12,27)	(-1,1),(-4,1);(-1,0),(-4,0)
(2328,112293)	(9,2080),(-14,2080);(9,-2079),(-14,-2079)
(-48,27)	(-2,1),(-3,1);(-2,0),(-3,0)
(60,27)	(0,1),(-5,1);(0,0),(-5,0)

Table 2

Other points on E_7 : $(u, \pm v) = (5280, 383643),\ (20616, 2960091)$.

(8)	$P_6(x) = \binom{y}{4}$
Transformation:	$u = 6x^2 + 30x + 20$ $v = 3y^2 - 9y + 3$
Elliptic curve:	E_8: $u^3 - 3024u - 33831 = v^2$
Integer points on E_8: $(u, \pm v) =$	Corresponding integer points on (8): $(x, y) =$
(-4,3)	(-1,0),(-1,3),(-4,0),(-4,3);(-1,1),(-1,2),(-4,1),(-4,2)
(-16,3)	(-2,0),(-2,3),(-3,0),(-3,3);(-2,1),(-2,2),(-3,1),(-3,2)
(20,3)	(0,0),(0,3),(-5,0),(-5,3);(0,1),(0,2),(-5,1),(-5,2)

Table 3

Other points on E_8 : $(u, \pm v) = (442124, 293978883),\ (62, 465),\ (50, 327),\ (2312, 111165),$

$$(-10,33),\ (39,212),\ (4244,276477),\ (-9,32),\ (51,338),\ (5216,376707),$$

$$(20696,2977347),\ (150,1823),\ (464,9987),\ (-13,30),\ (26,87),$$

$$(24,67),\ (110,1137),\ (-6,23),\ (179,2382),\ (212,3075),\ (5879,450768).$$

(9)	$\binom{x}{3} = P_2(y)$
Transformation:	$u = 6x - 6$ $v = 36y + 18$
Elliptic curve:	E_9: $u^3 - 36u + 324 = v^2$
Integer points on E_9: $(u, \pm v) =$	Corresponding integer points on (9): $(x, y) =$
(6,18)	(2,0);(2,-1)
(210,3042)	(36,84);(36,-85)
(-6,18)	(0,0);(0,-1)
(0,18)	(1,0);(1,-1)
(30,162)	(6,4);(6,-5)
(42,270)	(8,7);(8,-8)
(1224,42822)	(205,1189);(205,-1190)

Table 4

Other points on $E_9: \ (u, \pm v) = \ (-8,10),\ (9,27),\ (16,62),\ (1,17).$

(10)	$\binom{x}{3} = P_4(y)$
Transformation:	$u = 6x - 6$ $v = 36y^2 + 108y + 36$
Elliptic curve:	E_{10}: $u^3 - 36u + 1296 = v^2$
Integer points on E_{10}: $(u, \pm v) =$	Corresponding integer points on (10): $(x, y) =$
(54,396)	(10,2),(10,-5);–
(-6,36)	(0,0),(0,-3);(0,-1),(0,-2)
(0,36)	(1,0),(1,-3);(1,-1),(1,-2)
(6,36)	(2,0),(2,-3);(2,-1),(2,-2)
(384,7524)	(65,13),(65,-16);–

Table 5

Other points on $E_{10}: \ (u, \pm v) = \ (-12,0),\ (13,55),\ (150,1836),\ (21,99),\ (10,44),$

$$(36,216),\ (1066,34804),\ (138,1620),\ (-11,19),\ (82656,23763564)\ .$$

(13)	$\binom{x}{6} = P_2(y)$
Transformation:	$u = 45x^2 - 225x + 150$ $v = 8100y + 4050$
Elliptic curve:	E_{13}: $u^3 - 18900u + 15862500 = v^2$
Integer points on E_{13}: $(u, \pm v) =$	Corresponding integer points on (13): $(x, y) =$
(150,4050)	(0,0),(5,0);(0,-1),(5,-1)
(-120,4050)	(2,0),(3,0);(2,-1),(3,-1)
(-30,4050)	(1,0),(4,0);(1,-1),(4,-1)
(2400,117450)	(10,14),(-5,14);(10,-15),(-5,-15)
(3120,174150)	(11,21),(-6,21);(10,-22),(-5,-22)

Table 6

Other points on E_{13} : $(u, \pm v) =$ $(870, 25650)$, $(-264, 1566)$, $(1905, 83025)$, $(249, 5157)$, $(8250, 749250)$, $(64, 3862)$, $(366, 7614)$, $(330, 6750)$, $(-255, 2025)$, $(130, 3950)$, $(159720, 63832050)$, $(25, 3925)$, $(-174, 3726)$, $(600, 14850)$, $(1014, 32238)$, $(21030, 3049650)$, $(158505, 63105075)$, $(5470, 404450)$, $(10914707400, 1140297432700050)$.

(14)	$\binom{x}{6} = P_4(y)$
Transformation:	$u = 45x^2 - 225x + 150$ $v = 8100y^2 + 24300y + 8100$
Elliptic curve:	E_{14}: $u^3 - 36u + 324 = v^2$
Integer points on E_{14}: $(u, \pm v) =$	Corresponding integer points on (14): $(x, y) =$
(150,8100)	(0,0),(5,0),(0,-3),(5,-3);(0,-1),(5,-1),(0,-2),(5,-2)
(-120,8100)	(2,0),(3,0),(2,-3),(3,-3);(2,-1),(3,-1),(2,-2),(3,-2)
(-30,8100)	(1,0),(4,0),(1,-3),(4,-3);(1,-1),(4,-1),(1,-2),(4,-2)

Table 7

Other points on E_{14} : $(u, \pm v) =$ $(-291, 6777)$, $(3570, 213300)$, $(7980, 712800)$, $(32550, 5872500)$, $(61, 8009)$.

(15)	$\binom{x}{6} = \binom{y}{2}$
Transformation:	$u = 90x^2 - 450x + 300$ $v = 16200y - 8100$
Elliptic curve:	E_{15}: $u^3 - 75600u + 61290000 = v^2$
Integer points on E_{15}: $(u, \pm v) =$	Corresponding integer points on (15): $(x, y) =$
(300,8100)	(0,1),(5,1);(0,0),(5,0)
(-240,8100)	(2,1),(3,1);(2,0),(3,0)
(-60,-8100)	(1,0),(4,0);(1,1),(4,1)
(840,-24300)	(-1,2),(6,2);(-1,1),(6,-1)
(2460,121500)	(-3,8),(8,8);(-3,-7),(8,7)
(4800,-332100)	(-5,-20),(10,-20);(-5,21),(10,21)
(11640,1255500)	(-9,78),(14,78);(-9,-77),(14,-77)

Table 8

Other points on E_{15} : $(u, \pm v) =$ $(-456, 972)$, $(80436, -22812516)$, $(516, -12636)$,

$(370585, 225596125)$, $(1785, -74925)$, $(8400, 769500)$, $(136, 7316)$, $(-375, 6075)$,

$(84, -7452)$, $(160, 7300)$.

(16)	$\binom{x}{6} = \binom{y}{4}$
Transformation:	$u = 30x^2 - 150x + 100$ $v = 900y^2 - 2700y + 900$
Elliptic curve:	E_{16}: $u^3 - 8400u + 650000 = v^2$
Integer points on E_{16}: $(u, \pm v) =$	Corresponding integer points on (16): $(x, y) =$
(100,900)	(0,0),(5,0),(0,3),(5,3);(0,1),(5,1),(0,2),(5,2)
(-80,900)	(2,0),(3,0),(2,3),(3,3);(2,1),(3,1),(2,2),(3,2)
(-20,900)	(1,0),(4,0),(1,3),(4,3);(1,1),(4,1),(1,2),(4,2)
(280,4500)	(6,4),(-1,4),(6,-1),(-1,-1);—
(1600,63900)	(10,10),(-5,10),(10,-7),(-5,-7);—

Table 9

Other points on E_{16} : $(u, \pm v) =$ $(80, 700)$, $(6220, 490500)$, $(-56, 972)$, $(145, 1575)$,

$(2780, 146500)$, $(196, 2556)$, $(1000, 31500)$, $(56, 596)$, $(1220, 42500)$, $(-116, 252)$,

$(25, 675)$, $(316, 5436)$, $(520, 11700)$, $(20, 700)$, $(1, 801)$, $(-100, 700)$, $(64, 612)$,

$(6580, 533700)$, $(8081225, 22972898725)$, $(2261, 3415779)$, $(75580, 20778300)$.

(17)	$S_2(x) = P_2(y)$
Transformation:	$u = 12x + 6$ $v = 72y + 36$
Elliptic curve:	E_{17}: $u^3 - 36u + 1296 = v^2$
Integer points on E_{17}: $(u, \pm v) =$	Corresponding integer points on (17): $(x, y) =$
(54,396)	(4,5);(4,-6)
(-6,36)	(-1,0);(-1,-1)
(150,1836)	(12,25);(12,-26)
(6,36)	(0,0);(0,-1)
(138,1620)	(11,22);(11,-23)

Table 10

Other points on E_{17} : $(u, \pm v) =$ $(-12, 0)$, $(13, 55)$, $(21, 99)$, $(0, 36)$, $(10, 44)$, $(36, 216)$,

$(-11, 19),\ (384, 7524).$

(18)	$S_2(x) = P_4(y)$
Transformation:	$u = 12x + 6$ $v = 72y^2 + 216y + 72$
Elliptic curve:	E_{18}: $u^3 - 36u + 5184 = v^2$
Integer points on E_{18}: $(u, \pm v) =$	Corresponding integer points on (18): $(x, y) =$
(6,72)	(0,0),(0,-3);(0,-1),(0,-2)
(-6,72)	(-1,0),(-1,-3);(-1,-1),(-1,-2)

Table 11

Other points on E_{18} : $(u, \pm v) =\ (-18, 0),\ (21, 117),\ (144, 1728),\ (60, 468),\ (0, 72),$

$(34, 208),\ (570, 13608),\ (582, 14040).$

(21)	$S_5(x) = P_2(y)$
Transformation:	$u = 6x^2 + 6x - 1$ $v = 36y + 18$
Elliptic curve:	E_{21}: $u^3 - 3u + 322 = v^2$
Integer points on E_{21}: $(u, \pm v) =$	Corresponding integer points on (21): $(x, y) =$
(-1,18)	(0,0),(-1,0);(0,-1),(-1,-1)

Table 12

Other points on E_{21} : $(u, \pm v) =\ (17, 72),\ (2, 18),\ (9, 32),\ (-7, 0),\ (143, 1710).$

(22)	$S_5(x) = P_4(y)$
Transformation:	$u = 6x^2 + 6x - 1$ $v = 36y^2 + 108y + 36$
Elliptic curve:	E_{22}: $u^3 - 3u + 1294 = v^2$
Integer points on E_{22}: $(u, \pm v) =$	Corresponding integer points on (22): $(x, y) =$
(-1,36)	(0,0),(-1,0),(0,-3),(-1,-3);(0,-1),(-1,-1),(0,-2),(-1,-2)

Table 13

Other points on E_{22} : $(u, \pm v) =\ (2, 36),\ (47, 324),\ (-10, 18),\ (15, 68).$

(23)	$S_5(x) = \binom{y}{2}$
Transformation:	$u = 12x^2 + 12x - 2$ $v = 72y - 36$
Elliptic curve:	E_{23}: $u^3 - 12u + 1280 = v^2$
Integer points on E_{23}: $(u, \pm v) =$	Corresponding integer points on (23): $(x, y) =$
(-2,36)	(0,1),(-1,1);(0,0),(-1,0)
(22,108)	(1,2),(-2,2);(1,-1),(-2,-1)
(862,25308)	(8,352),(-9,352);(8,-351),(-9,-351)
(142,1692)	(3,24),(-4,24);(3,-23),(-4,-23)

Table 14

Other points on E_{23} : $(u, \pm v) =$ $(61, 477)$, $(4, 36)$, $(-10, 20)$, $(16, 72)$, $(-11, 9)$, $(38, 236)$.

3. Description of the algorithm

In this final section we will briefly outline the algorithm used to obtain our tables. As an example, we deal with equation (15) of Table 1, and follow the discussion and terminology of [GPZ] without any further referation.

Let, as above,

$$E_{15} : u^3 - 75600u + 61290000 = v^2 \ \cup \{\mathcal{O}\} \ ,$$

where $\mathcal{O}$ denotes the point at infinity. In the sequel we determine some parameters of E_{15} using SIMATH.

The modular invariant of E_{15} is

$$j = \frac{j_1}{j_2} = \frac{-1404928}{46899} \ ,$$

and the height of E_{15} is

$$\mu_\infty = 5.97704241... \ .$$

To use the algorithm of [GPZ], one has to know a basis as well as the torsion group of E_{15}. Using SIMATH, it turns out that the only torsion point of E_{15} is $\mathcal{O}$. The rank of E_{15} is $r = 2$, and a basis of the Mordell-Weyl group of E_{15} is $\{P_1 = (300, 8100),\ P_2 = (-240, 8100)\}$ with

$$\hat{h}(P_1) = 0.42722736...,\ \hat{h}(P_2) = 0.44856058... \ ,$$

where $\hat{h}(.)$ is the Néron-Tate height. It is well known that $\hat{h}(.)$ is a positive semidefinite quadratic form. For the smallest eigenvalue λ_1 of $\hat{h}(.)$ we obtain $\lambda_1 = 0.24519249...$.

The real and complex periods of E_{15} are

$$\omega_1 = 1.33636708...$$

and

$$\omega_2 = 0.66818354... + i \cdot 0.34901212... ,$$

respectively. Put $\tau = \omega_2/\omega_1$; thus

$$Im(\tau) = 0.26116486... .$$

Set $c_1 = \max\{\frac{\log(2^{13/6}/\omega_1)}{\lambda_1}, 1\}$; we have $c_1 < 4.94254024$. Moreover, with the notation $h = \log(11497758840000)$ we have

$$\max\left\{\hat{h}(P_i), h, \frac{3\pi|u_i|^2}{\omega_1^2 Im(\tau)}\right\} = h \text{ for } i = 1, 2 .$$

Let

$$V_i = e^h = 11497758840000 \text{ for } i = 1, 2 .$$

We define

$$c_2 := \max\left\{\frac{C}{\lambda_1}, 10^9\right\}\left(\frac{h}{2}\right)^3 \prod_{i=1}^{2} \log V_i$$

with the constant $C = 1.1 \cdot 10^{7r+9} \cdot (2/e)^{2r^2} \cdot (r+1)^{4r^2+10r} < 1.42 \cdot 10^{39}$. Substituting the values of the parameters into the above formula, we obtain

$$c_2 < 4.48 \cdot 10^{48} .$$

Let now $P = n_1 P_1 + n_2 P_2$ be an integer point on E_{15} with $n_1, n_2 \in \mathbb{Z}$ and put $N = \max\{|n_1|, |n_2|\}$. Using the above parameters, the Theorem of [GPZ] yields the estimate

$$N \leq \max\left\{N_1, \frac{2V}{r+1}\right\}$$

with $V = \max\{V_1, V_2\}$ and $N_1 = 2^{r+2}\sqrt{c_1 c_2}(\log(c_2(r+2)^{r+2}))^{(r+2)/2}$. That is, we obtain the initial bound

$$N < 1.05 \cdot 10^{30} .$$

Now we will use B. M. M. de Weger's method (see [dW1]) to reduce this bound. We will outline de Weger's method in our special case only.

Consider the 3×3 matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ [C_0 u_1] & [C_0 u_2] & C_0 \end{pmatrix}$$

where u_1 and u_2 are the elliptic logarithms of P_1 and P_2, respectively. Using SIMATH again, we obtain

$$u_1 = 0.24604231... \text{ and } u_2 = 0.40147506... .$$

Let b_1, b_2, b_3 be the LLL-reduced basis of the lattice spanned by the coloumns of the above matrix. We set

$$N' = \frac{1}{2\sqrt{18}}\, ||b_1|| .$$

Now if $N' > N$, then we have the new estimate

$$N \leq \sqrt{\frac{1}{\lambda_1} \log \frac{2^{\frac{6}{7}} C_0}{\omega_1 N'}} \,.$$

We start with $C_0 = 10^{91}$, and we get $N' = 2.65 \cdot 10^{60}$, whence $N \leq 17.02$. Now we repeat the whole process with $C_0 = 10^7$ to obtain $N' = 21.81$ and $N \leq 7.44$. A third application of de Weger's method yields the same bound for N. We now only have to test the integrality of the points

$$n_1 P_1 + n_2 P_2 \text{ with } |n_1|, |n_2| \leq 7.$$

We have to check the pairs

$$(u, v) \in \mathbb{Z}^2 \text{ with } \log |u| \leq \mu_\infty = 5.97704241...$$

as well as the case $u < 0$ (cf. [GPZ]). After all, we obtain that the only integral points on E_{15} are those given in Table 8. $\square$

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