Probability theory and mathematical statistics
Excercises 1.

1. A coin is tossed. If the result is a head, it is tossed once again, otherwise it is tossed twice again. Give the sample space of the experiment. What is the probability that only one of the tosses resulted a head?

2. Give the sample space of the five from ninety lottery.

3. A dice is thrown three times. Let A_i denote the event that the result of the ith throw is six, $i = 1, 2, 3$. What is the meaning of the following events:

$$A_1 + A_2 \ , \ A_1 \cdot A_2 \ , \ A_1 + A_2 + A_3 \ , \ A_1 \cdot A_2 \cdot A_3 \ , \ A_1 \cdot \overline{A_2} \ , \ A_1 \backslash A_2,$$

4. In a workshop there are three machines. Let A_i denote the event that the ith machine breaks down in a year, $i = 1, 2, 3$. With the help of events A_i express the following statements:

   (a) only the first breaks down;
   (b) all three break down;
   (c) none of the machines break down;
   (d) the first and the second do not break down;
   (e) the first and the second break down, the third does not;
   (f) only one machine breaks down;
   (g) at most one machine breaks down;
   (h) at most two machines break down;
   (i) at least one machine breaks down.

5. A fair dice is thrown. What is the probability that we get

   (a) at least four;
   (b) at most four;
   (c) an even number;
   (d) an odd number less than five;
   (e) an even number greater than two or an odd number less then five?

6. Two fair dice are thrown. Find the probability that the sum of the numbers obtained is 8. Illustrate the sample space and the set of favourable events.

7. Three fair dice are thrown. Find the probability that the sum of the numbers obtained is 9.

8. A fair dice is thrown six times. What is the probability that
   (a) we obtain all six numbers at one time;
   (b) at the first throw we obtain 6, and the other results are different from that;
   (c) the first and the second throw obtain 6, and the other throws obtain different from 6 and from each other;
   (d) we obtain two sixes, and the other obtained numbers are different from that;
   (e) we don't obtain any sixes;
   (f) all throws obtain even numbers?
9. Four fair coins are tossed. What is the probability that we will have
   (a) four heads;
   (b) two heads and two tails;
   (c) at most one head?
10. In an urn we have 4 red, 3 white and 2 green balls. We choose two balls randomly together. What is the probability of them having the same colour?
11. What is the probability that two persons in a group of four have their birthdays on the same day (365 day of a year considered)?
12. From 100 bananas 10 are rotten. What is the probability of having exactly one rotten among five randomly chosen bananas?
13. In an urn we have 20 red and 30 white balls. 10 balls are chosen without replacement. Find the probability that
   (a) all the chosen balls are red;
   (b) 4 red, 6 white;
   (c) at least one red?
14. Solve the previous excercise under the assumption that the balls are chosen with replacement.
15. Find the probability that on the lottery 5 from 90 we hit at least three winning numbers.
16. We place 8 different objects in 3 boxes. What is the probability that we placed 2 objects in the first box, 4 objects in the second box, and 2 objects in the third box?
17. Nine people get on a three wagon train randomly. What is the probability that in all wagons there are three of them?