Probability theory and mathematical statistics
Excercises 3.

1. Consider a family with two children. Assume that each child is as likely to be a boy as it is to be a girl. What is the probabilty that both children are boys, given that

   (a) the older child is a boy;
   (b) at least one of the children is a boy.

2. Three dice are rolled. What is the probability that one of the numbers obtained is 6 given that the sum is 12?

3. Mosquitoes are usually killed using sprayed chemicals. Scientists found that the first treatment kills 80% of the mosquitoes. However, the surviving insects become resistant, so the second treatment kills 40%, while the third kills just 20% of the mosquitoes.

   - What is the probability that a mosquito survives all three treatments?
   - Given a mosquito survived the first treatment, what is the probability that it survives two more treatments?

4. At a university exam Mathematics, Physics and Informatics students take part, 20 students each. 60% of the Mathematics, 80% of the Physics and 50% of the Informatics students are successful at the exam. What is the probability that a randomly chosen student is successful?

5. There are six six-shooter revolvers laying on a table. Three of them are loaded with $1-1$ bullets, two of them with $2-2$ bullets, and the sixth with 3 bullets. We chose randomly a revolver and pull the trigger. Find the probability that the chosen revolver shots.

6. A box contains 3 blue and 2 red marbles while another box contains 2 blue and 5 red marbles. A marble drawn at random from one of the boxes turns out to be blue. What is the probability that it came from the first box?

7. Rust Rider cars are produced in four factories. The first factory produces 200 cars per day, the second 320, the third 270 while the fourth 210. The refuse ratios for the factories are 2%, 5%, 3% and 1%, respectively. We bought a Rust Rider and we found it perfect. What is the probability that it had been produced in the fourth factory?

8. In an office equipped with mechanized administration three machines classify the files. The first can process 10 files per day, the second 15, while the third 25. The average numbers of misclassified files are 0.3; 0.9 and 0.5 per day, respectively. We choose a file randomly from the daily production and we find that it has been misclassified. What is the probability that the file was processed by the first machine?

9. Each of three identical jewelry boxes has two drawers. In each drawer of the first box there is a gold watch. In each drawer of the second box there is a silver watch. In one drawer of the third box there is a gold watch while in the other there is a silver watch. If we select a box at random, open one of the drawers and find it to contain a silver watch, what is the probability that the other drawer has the gold watch?

10. On a multiple-choice test three answers correspond to each question and only one answer is correct. A student taking the test knows the correct answer with probability p. Otherwise, he chooses an answer randomly, i.e. with probability $\frac{1}{3}$. Given his answer is correct, what is the probability that he knew the right answer and not only guessed?