Probability theory and mathematical statistics
Excercises 5.

1. We shoot on a round target having unit radius. Assume that each shot hits the target and the location of the hit is uniformly distributed on the target. Let ξ denote the distance of the hit from the center of the target. Give the commulative distribution function (cdf), probability density function (pdf), expected value and standard deviation of ξ.

2. Check whether the following functions are cumulative distribution functions or not.

    (a)
    $$F(x) = \begin{cases} 0 & \text{if } x < 0, \\ \frac{x}{x+1} & \text{if } x \geq 0. \end{cases}$$

    (b)
    $$F(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ 1 & \text{if } x > 0. \end{cases}$$

    (c)
    $$F(x) = \begin{cases} 0 & \text{if } x < 0, \\ 2 & \text{if } 0 \leq x < 1. \\ 1 & \text{if } x \geq 1. \end{cases}$$

3. Check whether the following functions are probability density functions (pdf) or not.

    (a)
    $$f(x) = \begin{cases} \frac{\sin x}{2} & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

    (b)
    $$f(x) = \begin{cases} \frac{1}{x^2} & \text{if } x > 1, \\ 0 & \text{otherwise.} \end{cases}$$

    (c)
    $$f(x) = \begin{cases} \frac{x}{x+1} & \text{if } 0 < x < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

(d)
$$f(x) = \begin{cases} \frac{1}{3} & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

(e)
$$f(x) = \frac{1}{\pi(1 + x^2)}$$

4. The cdf of a random variable is
$$F(x) = \begin{cases} 0 & \text{if } x \leq 1, \\ (x-1)^3 & \text{if } 1 < x \leq 2, \\ 1 & \text{if } x > 2. \end{cases}$$

Find the pdf, the expected value and the variance of the variable.

5. The pdf of a random variable ξ equals
$$f(x) = \begin{cases} \frac{2}{3} & \text{if } 0 \leq x < 1, \\ \frac{1}{3} & \text{if } 1 \leq x < 2, \\ 0 & \text{otherwise.} \end{cases}$$

Find the cdf, the expected value and the variance of the variable.

6. The pdf of a random variable ξ equals
$$f(x) = \begin{cases} 0 & \text{if } x < 0, \\ cx^2 & \text{if } 0 \leq x \leq 2, \\ 0 & \text{if } 2 < x. \end{cases}$$

Find the value of c, the cdf of ξ, the probability $P(1 < \xi < 3)$, the expected value and standard deviation of ξ.

7. The pdf of a random variable ξ equals
$$f(x) = \begin{cases} 0 & \text{if } x \leq 2, \\ \frac{a}{x^3} & \text{if } x > 2. \end{cases}$$

Find the value of a, the cdf, the expected value and standard deviation of ξ. For what x does $P(\xi > x) = \frac{1}{2}$ hold?

8. The pdf of a random variable ξ equals
$$f(x) = \begin{cases} 0 & \text{if } x \leq 0, \\ \frac{1}{\sqrt{x}} & \text{if } 0 < x \leq \frac{1}{4}, \\ 0 & \text{if } \frac{1}{4} < x. \end{cases}$$

Find the cdf, the expected value and standard deviation of ξ. What is the probability of the event that the difference of ξ and 0 is less then 0.1?

9. A point is chosen randomly on the interval $[0, a]$. Let ξ denote the distance of the point from the center of the interval. Find the cdf and pdf of ξ.

10. Choose a point inside a unit square randomly. Let ξ denote the distance of the chosen point and the nearest side of the square. Find the cdf, expected value and standard deviation of ξ.