Probability theory and mathematical statistics
Excercises 6.

1. We toss a fair coin three times. Find the probability distribution of the number of heads appeared. Find the expected value and the standard deviation of the number of heads. What is the probability of more then 1 heads appearing?

2. We toss a fair dice five times. Find the probability distribution of the number of sixes appeared. Find the expected value and the standard deviation of the number of sixes. What is the probability of throwing at least 3 sixes?

3. We toss a fair coin 100 times. What is the expected number of heads? What is the standard deviation?

4. Out of 2000 families with 4 children each, what is the expected number of families having at least 1 boy?

5. If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random, less then 2 bolts will be defective.

6. Find the probability of getting a total of 7 at least once in three tosses of a pair of fair dice.

7. In an urn we have 4 brown and 5 green balls. We choose 3 balls. ξ is the number of chosen brown balls. Find the probability distribution, expected value and standard deviation of ξ.

8. A box contains 5 red and 10 white marbles. If 8 marbles are to be chosen at random (without replacement), determine the probability that (a) 4 will be red; (b) all will be white; (c) at least one will be red.

9. A phone call center receives 4 calls per minute on an average day. What is the probability that they receive 6 calls in 1 minute?

10. In az average 400 page book there are 400 mistakes. What is the probability that on a randomly chosen page there at least 3 mistakes?

11. Let ξ be uniformly distributed in $[-2, 2]$. Find $P(\xi < 1)$ and $P\,|\xi - 1| \geq \frac{1}{2}$.

12. Let ξ be an uniformly distributed random variable with $\mathbb{E}\xi = \mathbb{D}^2\xi = 4$. Give the cdf and pdf of ξ.

13. The probability that at a filling station one has to wait more than 6 minutes for the service is 0.1. Given the waiting time is exponentially distributed find the probability that we are served in 3 minutes after arrival?

14. The life-span of a plasma TV is an exponentially distributed random variable with a mean life-span of 9 years. Find the largest number K such that the probability that the TV is operable for K years is at least 0.9.

15. At a gas station the average waiting time is 4 minutes. If the waiting time is exponentially distributed, what is the probability of waiting more than 3 minutes but less than 4 minutes?

16. If ξ is normally distributed with expected value 5 and standard deviation 2, find $P(X > 8)$.

17. An automatic packing machine fills cartons with 1 kilogram of sugar. The amount of sugar filled into a carton is a normal random variable with mean 1 kg and standard deviation 32g. The quality control accepts a carton if its weight is between 0.95 and 1.05 kilograms. Find the probability that a randomly chosen carton is accepted.

18. A sawmill produces boards. The length of boards, in centimetres, is normally distributed with mean 400 and standard deviation 3. Find the percentage of boards that are longer than 398 cm and shorter than 401 cm. Find the probability that the length of a randomly chosen board differs from the mean with at most 2.5 cm.

19. A cannon shoots on a target at a distance of 1200 metres. The deviation of the length of a shot around the 1200 m is a normal random variable with a standard deviation 40 m. A shot is efficient if the hit is closer to the target than 50 m. Find the percentage of inefficient shots.

20. The air control informs the pilot of an aircraft about the altitude of the centre of the air corridor of height 100 metres where the aircraft should fly. The deviation, in metres, of the altitude of the aircraft from the given altitude is normally distributed with mean 20 and standard deviation 50. Find the probability that the aircraft flies under the air corridor, in the air corridor, above the air corridor.

21. The diameter, in centimetres, of bearings produced by a machine is normally distributed with mean 15 and standard deviation 0.5. Find the probability that the difference in the diameter of a bearing produced by this machine is more than 5% of the required value.

22. The number ξ of calls arriving to a certain break-down assistance can be considered as a normal random variable with a standard deviation $\sigma = 10$. What is the mean number of calls if $P(\xi < 20) = 0.1$?

23. The Light Bulb Company produces fluorescent lamps. The life-span of the lamps follows normal distribution with mean 1170 hours and standard deviation 100 hours. The company gives warranty on the lamps and during the warranty period it replaces a dead lamp free of charge. How long the warranty period should last if the company wants to replace at most 5% of the sold lamps.