Probability theory and mathematical statistics
Excercises 7.

1. The following table specifies the joint distribution of (ξ, η).

ξ \ η	-1	0	1
-1	p	$3p$	$6p$
1	$5p$	$15p$	$30p$

   - Find the value of p;
   - find the marginal probability distribution of ξ and η;
   - find the joint cdf and the marginal distribution functions of ξ and η;
   - find the distributions of $\xi + \eta$ and $\xi \cdot \eta$;
   - find the expected values of ξ, η, $\xi + \eta$, $\xi \cdot \eta$;
   - find the covariance and the correlation coefficient of ξ and η.

2. The joint cdf of (ξ, η) equals

$$F(x,y) = \begin{cases} 0 & \text{if } x \le 1 \text{ or } y \le 1, \\ 1 + \frac{2}{x+y} - \frac{2}{1+x} - \frac{2}{1+y} & \text{otherwise.} \end{cases}$$

   Find the marginal distribution functions of ξ and η, the joint pdf, the marginal density functions and the probability $P(0 \le \xi < 3, 1 \le \eta < 2)$.

3. Let the joint pdf of (ξ, η) be

$$f(x,y) = \begin{cases} \frac{1}{3}(x+y) & \text{if } 0 < x < 1 \text{ and } 0 < y < 2, \\ 0 & \text{otherwise.} \end{cases}$$

   Find the joint cdf, the marginal distribution functions, the marginal density functions, the expected values of ξ, η, $\xi + \eta$, $\xi \cdot \eta$ and the covariance and correlation coeffitient of ξ and η.

4. Let the joint pdf of (ξ, η) be

$$f(x,y) = \begin{cases} 2 & \text{if } 0 < x < 1 \text{ and } 0 < y < x, \\ 0 & \text{otherwise.} \end{cases}$$

   Find the pdf of $\xi + \eta$!

5. Let ξ on $[0, 1]$, and η on $[2, 4]$ be uniformly distributed, independent random variables. Find the cdf and pdf of $\xi + \eta$.

6. Let ξ and η be uniformly distributed independent random variables on $[0, 1]$. Find the pdf of $\xi - \eta$, and the probability of $|\xi - \eta| < \frac{1}{4}$.