Annales Univ. Sci. Budapest., Sect. Comp. 40 (2013) 9–20

LAUDATION TO ZOLTÁN DARÓCZY

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1. The life of Zoltán Daróczy

Zoltán Daróczy was born in Bihartorda, on the 23rd of June, 1938. His father, dr. Bálint Daróczy was a county recorder, and his mother, Ilona Jónás was a housewife.

Zoltán, his parents and his sister and brother

This paper was realized in the frames of TÁMOP 4.2.4. A/2-11-1-2012-0001 National Excellence Program – Elaborating and operating an inland student and researcher personal support system The project was subsidized by the European Union and co-financed by the European Social Fund. The preparation of this paper was also supported by the Hungarian Scientific Research Fund (OTKA) Grant NK 81402.

He had a brother, Sándor, who was a physicist, and a younger sister, Éva, a chemist. During the 2nd World War the family was on the run, they spent their time in a cellar, then in February, 1945, they were able to leave their shelter.

Zoltán, his sister Éva and his brother Sándor

He finished his primary school in Báránd. In 1952 he was not allowed to enter the secondary school at Püspökladány. Probably the reason was that his father had been considered ”suspicious”, since he was an attorney. Hence he started to continue his studies in the Protestant College of Debrecen. He spent there four years and got his maturation with distinction.

In that college his master was Géza Nagy, the teacher of mathematics and physics, who inspired Zoltán to participate in the mathematical competitions. Zoltán succeeded at the competitions Dániel Arany, at Mathematical Journal for Secondary Schools (KÖMAL), and later at the competition named after Mátyás Rákosi. He was interested in several things that time, such as literature, history, chess, athletics, etc.

He wanted to start his university studies in 1956, the year of the Hungarian Revolution. Although he was not accepted, but his chess-abilities helped him to enter the university. His major subjects were mathematics and descriptive geometry.

As everyone knows, the year 1956, especially its autumn, was a very important period in the history of Hungary. That time Zoltán was a member of the Hungarian National Guard of the University which capitulated to the Russian army – and even so they were not deported.

He graduated from Debrecen University in 1961 with distinction. He got married in 1960 and his first son, Zoltán jr. was born.

After his graduation, he could not find any position in Debrecen. Fortunately, in 1961–62 he obtained a special scholarship from the Hungarian Academy of Sciences, which was formally paid by Alfréd Rényi, however, the work was given by János Aczél. The fruit of this scholarship was a paper, published in Acta Sci. Math. Szeged.

Zoltán told us the following story: János Aczél asked Béla Szőkefalvi-Nagy about the problem: "does there exist a nonzero, additive and microperiodic function?" He enjoyed it very much that Szőkefalvi had not believed in the existence of such a function. However, Zoltán figured out the following very simple construction: take a Hamel base H (of $\mathbb{R}$ over $\mathbb{Q}$), a fixed element h in H, and define $f(h) = 0$, further let $f(u) \neq 0$ for every other element u in H. Then, for each rational number r, f is periodic with rh and hence it is microperiodic.

He got a lecturer position in 1962 in Debrecen, and in that year, he also received his PhD. In 1963–64 he got an Austrian scholarship in Vienna, where he worked under the supervision of L. Schmetterer. In 1967 he defended his academic dissertation, then in 1968 he was promoted as Associate Professor, and he became the chairman of the Department of Analysis.

Starting from 1969, he was the vice dean of the Faculty of Sciences for five years, and from 1974 he was elected to be the dean for six years. In 1976, he defended his *Doctor of Science* dissertation and became a full professor at Kossuth Lajos University.

With his wife Erzsébet

In 1981 he divorced. Later he married Erzsébet Kotora. From this marriage in 1983 his second son, Bálint, and in 1986 his daughter, Orsolya was born. He has three grandchildren: Ádám (1990) and Péter (1995) from his first son Zoltán, and Sára (2013) from his second son Bálint.

Starting from 1984 he was the vice rector of the Kossuth Lajos University, and the director of the Mathematical Institute. From 1985 he became a corresponding member and in 1990 he was elected to be a full member of the Hungarian Academy of Sciences.

During the period from 1987 to 1990 he was the rector of the Kossuth Lajos University, and after it he withdrew from the leading positions in the Mathematical Institute. From 1992 he was the leader of the PhD program and then the Doctoral School of Mathematical and Computational Science until 2008 when he was retired.

His political activity started in 1967, it lead to a position of "Member of the Parliament". At the first free elections in 1990 and in 1994 he was a representative of the city of Debrecen. He tried to serve education and similar fields. There are two parliamentary sayings which can be attached to his name: about the compensation, he said "I see no aim, no sense, but no obstacle". And about the academy: "A modern state must keep his academicians and insanes."

His most important travels: to Pavia (hosted by Bruno Forte), with UNESCO in western Europe, 1-1 months in Nijmegen, Paderborn, and lots of conferences.

Giving a talk at one of the many conferences

He was decorated with the following distinctions: *Géza Grünwald Prize* of the Bolyai János Mathematical Society (1963); *Mathematical Prize* of the Hungarian Academy of Sciences (1979); *Tibor Szele Medal* of the Bolyai János Mathematical Society (1986); *Albert Szent-Györgyi Prize* of the Ministry of Education and Culture (1998); *Széchenyi Prize* (2004); *Middle Cross of the Hungarian Republic* (2008). He was also elected to be the Honorary Member of the Mathematical Society in Hamburg.

The school he has built up is existing and blossoming. His students' names are known all over the world. We have to mention some of them: László Losonczi is a "semi-student", because he was inspired by Zoltán to work with János Aczél. Károly Lajkó was inherited from Aczél. And then those he personally chose: Gyula Maksa, János Rimán, Antal Járai, László Székelyhidi, György Szabó, Zsolt Páles, Tamás Szabó, Gabriella Hajdu, István Blahota, Károly Nagy, Pál Burai, Judita Dăscal.

He was the PhD-leader of Béla Nagy. He managed Zoltán Sebestyén and János Demetrovics.

He expressed several times that he was proud of the young generation of the Analysis Department: Lajos Molnár, Zoltán Boros, Attila Gilányi, Mihály Bessenyei, Eszter Gselmann, Fruzsina Mészáros.

He is the guardian of the memory of his good old friends, Imre Makai, Ernő Gesztelyi, Jenő Erdős.

Outside of Debrecen his most important scientific contact is Imre Kátai, with whom he wrote several papers. It is an extraordinary occasion that they can celebrate their birthdays together.

Closing photo of the Debrecen-Katowice Winter Seminar in Tiszafüred

His other very influential friend is the pope of functional equations, János Aczél. Though they had been working together until they finished their book, later their fields of interest have developed into different branches. While working on their book with János Aczél, their connection was very intense. Though later they choose separate fields of mathematics, Zoltán is still grateful to János for his acknowledgement and support for the Debrecen school of functional equations.

As Madách, the famous Hungarian writer said, "the machine is working, the maker is having a rest". However, he does not follow Madách's words: he is as active as he can be.

2. Scientific works of Zoltán Daróczy

Now we turn to speak about Zoltán's mathematical achievements.

"Notwendige und hinreichende Bedingungen für die Existenz von nichkonstanten Lösungen linearer Funktionalgleichungen", published at Acta Sci. Math. Szeged, in 1961, an early paper. The problem discussed here can be formulated as follows: let $f:\mathbb{R}\to\mathbb{R}$ be a nonidentically zero additive function, and let $a\neq 0$, $p\neq 0$, $a\neq p$ be irrational numbers. What is the necessary and sufficient condition in order that

$$f(ax)=pf(x)$$

holds for all x in $\mathbb{R}$? The answer is: either a and p are the roots of the same irreducible polynomial over the rationals, or a and p are transcendental over the rationals.

There are several papers by him concerning functional equations involving composed functions. For instance the functional equation of Forte and Kampé de Fériet:

$$f[x+f(y)]+f(y)=f[y+f(x)]+f(x), \tag{1}$$

$$f(-x)=f(x)+x. \tag{2}$$

The following theorem holds true.

Theorem. *If $f:\mathbb{R}\to\mathbb{R}$ is a continuous solution of the system* (1) *and* (2), *then either*

$$f(x)=-\frac{1}{A}\log(1+e^{Ax}) \tag{3}$$

for all x in $\mathbb{R}$, or

$$f(x) = \pm\frac{1}{2}\{|x| \mp x\} \tag{4}$$

for all x in $\mathbb{R}$, where A is a nonzero constant.

Later on he settled the continuous solutions of the single equation (1) by proving that either f is a constant or it is a translate of a function of the form (3) or (4).

Something about information functions. He was an inventor on this field. His joint book with Aczél was very influential and served as a manual for those who were interested in the axiomatization of information theory. He calls a function $f : [0,1] \to \mathbb{R}$ an information function of β-type, if the following functional equation holds:

$$f(x) + (1-x)^{\beta} f\left(\frac{y}{1-x}\right) = f(y) + (1-y)^{\beta} f\left(\frac{x}{1-y}\right),$$

with the additional conditions $f(0) = f(1)$ and $f(\frac{1}{2}) = 1$, where β is a positive number, and x, y satisfy the conditions $0 \le x < 1$, $0 \le y < 1$, $x + y \le 1$.

If $\beta = 1$, then f is termed a standard information function.

He got the following theorem:

Theorem. *If $\beta \neq 1$ and $f : [0,1] \to \mathbb{R}$ is an information function of β-type, then*

$$f(x) = \frac{1}{2^{1-\beta} - 1}[x^{\beta} + (1-x)^{\beta} - 1],$$

and if β tends to 1, then this function tends to the so-called Shannon-information function defined by

$$S(x) = -x \log_2 x - (1-x)\log_2(1-x),$$

for each x in the open interval $]0,1[$, and is zero at the endpoints.

The solutions of this equation for $\beta = 1$ were also described by him, the noncontinuous ones can be expressed in terms of nonzero derivations.

He published numerous papers with Kátai. One of them deals with bounded information functions. They proved that if $f : [0,1] \to \mathbb{R}$ is an information function having the property $0 \le f(x) \le K$, then $f(x) = S(x)$, the Shannon information function.

If only nonnegativity is required, then the situation drastically changes. With Maksa he succeeded to prove the following result: if $f : [0,1] \to \mathbb{R}$ is an information function, which is nonnegative, then it is not less than the Shannon-function. The main idea here is that for an information function it is a very natural requirement that it should be nonnegative. A basic problem was: does there exist a nonnegative information function different from the Shannon one? The answer is "yes", and the general solution of the functional equation of information clears everything up: there are nonnegative information functions different from the Shannon one.

The theory of means, was one of his most frequently discussed and investigated topics. For a continuous and strictly monotonic function $\varphi :]0,1[\to \mathbb{R}$, he defined the n-variable mean

$$^{\varphi}M_n(p_1, p_2, \ldots, p_n) = \varphi^{-1}\left[\frac{\sum_{k=1}^{n} p_k \varphi(p_k)}{\sum_{k=1}^{n} p_k}\right]$$

where $p_1, p_2, \ldots, p_n$ form a so-called non-complete probability distribution. Then we have the statement:

Theorem. *If, for all $p_1, p_2, q > 0$ with $p_1 + p_2 \le 1$,*

$$^{\varphi}M_2(qp_1, qp_2) = q \cdot {}^{\varphi}M_2(p_1, p_2),$$

then either $\varphi(t) = \log t$, or $\varphi(t) = t^{\alpha}$ holds for all t in the interval $]0,1]$, up to an additive and a nonzero multiplicative constant, where $\alpha \neq 0$ is an arbitrary constant.

This was conjectured by Rényi, and Zoltán succeeded to prove it.

The next topic that we mention is a joint work with Aczél, about quasi-arithmetic means weighted by weight functions. These are related to the results of Bajraktarević. Here they dealt with the problem of equality of the means, and the characterization of homogeneous means of this class. His main contribution on this field was to find new mean values (partly together with Losonczi and Páles), which could be applied to define new classes of entropies, for instance.

On Hosszú's functional equation. The equation reads as follows:

$$f(x + y - xy) + f(xy) = f(x) + f(y),$$

where f goes from $\mathbb{R}$ to $\mathbb{R}$. Here his celebrated result is that the general solution is $f(x) = a(x) + b$, where a is additive, i.e., this functional equation is equivalent to the Jensen equation.

From the field of extension theorems we recall one result only, which is one of the most frequently used and cited result of Zoltán. This inspired the second author to write his first mathematical paper: on additive extensions of functions, which are additive on some open set in the plane. The idea of extension theorems goes back to Erdős, and Daróczy and Losonczi established an extension theorem in the case, where the domain of the original function was an open and connected set in the plane. This result had applications later – related to the work of Lajkó – concerning the restricted Jensen equation.

Our next subject is the theory of deviation means. Let I be an open interval in $\mathbb{R}$ and $E : I \times I \to \mathbb{R}$ be a two-variable function, which is continuous and strictly monotonically decreasing in the second variable. We suppose, in addition, that it vanishes along the diagonal: $E(y, y) = 0$. Then we call E a deviation. One can easily see that, for any n-tuple of elements $\underline{x} = (x_1, x_2, \ldots, x_n)$ of I, there exists a uniquely determined number $y = \mathcal{M}_E(\underline{x})$ with the property that it is between the minimum and the maximum of the variables (that means, it is a mean) and satisfies the condition:

$$\sum_{i=1}^{n} E(x_i, y) = 0.$$

The means defined as above are the so-called deviation means. Special cases are the arithmetic, geometric, harmonic, more generally quasi-arithmetic means. But the Bajraktarević means also form a subclass of deviation means.

The most basic question is the comparison problem for deviation mean. A deviation mean E is called differentiable, if its partial derivative with respect to its second variable is negative along the diagonal. We recall the main result concerning these concepts. Suppose that a real function f on $I \times I$ is given, and the partial derivatives with respect to both variables exist. Given a deviation E we denote

$$E^*(x, y) = \frac{E(x, y)}{-\partial_2 E(y, y)}.$$

We have the following theorem.

Theorem. *If H, K, L are differentiable deviations on I and $f : I \times I \to I$ is differentiable, then the inequality*

$$\mathcal{M}_H[f(\underline{x}, \underline{y})] \le f[\mathcal{M}_K(\underline{x}), \mathcal{M}_L(\underline{y})]$$

holds for all $\underline{x}, \underline{y}$ in I^n and for all n in $\mathbb{N}$ if and only if we have

$$H^*[f(u, v), f(t, s)] \le K^*(u, t) f_1(t, s) + L^*(v, s) f_2(t, s),$$

for all u, v, s, t in I.

This result implies Minkowski and Hölder type inequalities (with $f(x,y) = x + y$ and $f(x,y) = xy$, respectively). On the other hand, with $f(x,y) = x$, the comparison theorem for differentiable deviation means follows. The comparison problems for non-differentiable deviation means was established in a joint work with the first author of this paper.

Theorem. *Suppose that F, E are deviations. Then*

$$\mathcal{M}_E(\underline{x}) \leq \mathcal{M}_F(\underline{x}),$$

for all $\underline{x}$ in I^n and for all n in $\mathbb{N}$ if and only if we have

$$F(x,y)E(z,y) \leq F(z,y)E(x,y)$$

for any $x \leq y \leq z$.

And here comes the joint venture with Kátai, which is about additive number theoretical functions. He wrote about ten papers on this subject. In one of them the following problem is considered: given a compact Abelian group, we consider a function defined on the naturals, and having values in this group, with the property that it is a homomorphism. One can classify these functions: the first group is characterized by the property that the limit of their one-step difference is zero. The characterization of the second group is a little bit more complicated: it consists of all the functions φ, which have the property that if a subsequence $\{n_k\}$ of natural numbers exists with

$$\lim_{k\to\infty} \varphi(n_k) = g,$$

where $\{n_k\}$ is a strictly monotonic increasing sequence, then

$$\lim_{k\to\infty} \varphi(n_k + 1) = g.$$

The main theorem in this field is that the two above mentioned classes are the same.

Interval filling sequences was also in the focus of his interest in several papers. This attracted a number of young colleagues of him, who made their first steps on a scientific road. The next work is a joint one with Kátai and Járai. Take a strictly decreasing sequence $\{\lambda_n\}$ of positive numbers which has a finite sum, and we suppose that $\lambda_n \leq \sum_{i=n+1}^{\infty} \lambda_i$ for all n in $\mathbb{N}$. The statement is that any number x in the interval $[0, L]$ can be represented in the form

$$x = \sum_{n=1}^{\infty} \epsilon_n \lambda_n,$$

where $\epsilon_n = 0$ or 1 and $L = \sum_{i=1}^{\infty} \lambda_i$. By this property, such sequences are called interval filling sequences. There exist several algorithms to construct the digits ϵ_n. One of them is the eager, or regular algorithm, which is defined recursively by

$$\epsilon_n(x) = \begin{cases} 1, & \text{if } \sum_{i=1}^{n-1} \epsilon_i(x)\lambda_i + \lambda_n \le x \\ 0, & \text{if } \sum_{i=1}^{n-1} \epsilon_i(x)\lambda_i + \lambda_n > x, \end{cases}$$

then

$$x = \sum_{n=1}^{\infty} \epsilon_n(x)\lambda_n.$$

He kept on inventing and he arrived at the following concept. A function $F : [0, L] \to \mathbb{R}$ is called additive with respect to a sequence $\{\lambda_n\}$ if

$$\sum_{n=1}^{\infty} |F(\lambda_n)| < \infty\,,$$

and we have for all $x \in [0, L]$ that

$$F(x) = F\left(\sum_{n=1}^{\infty} \epsilon_n(x)\lambda_n\right) = \sum_{n=1}^{\infty} \epsilon_n(x)F(\lambda_n).$$

With Kátai they proved that there exists an interval filling sequence $\{\lambda_n\}$ and a continuous function $F : [0, L] \to \mathbb{R}$ additive with respect to $\{\lambda_n\}$, which is nowhere differentiable.

Another important result proved with Kátai and T. Szabó says that if F is additive with respect to an interval filling sequence for any algorithm, then F is linear.

Finally, we discuss his results concerning the invariance equation for means and the Matkowski–Sutô problem. If M, N, K are two-variable means on I, then we say that K is invariant with respect to the pair (M, N) if, for all x, y in I,

$$K(M(x, y), N(x, y)) = K(x, y).$$

It is well-known that if M and N are continuous strict means, then K is uniquely determined. The solution of this equation in the class of quasi-arithmetic means leads to the following equation, whose solution is called the Matkowski–Sutô problem:

$$\varphi^{-1}\Big(\frac{\varphi(x) + \varphi(y)}{2}\Big) + \psi^{-1}\Big(\frac{\psi(x) + \psi(y)}{2}\Big) = x + y. \tag{5}$$

(In other words, this equation says that the arithmetic mean is invariant with respect to two quasi-arithmetic means.) Matkowski, under twice continuous differentiability assumptions on the strictly increasing unknown functions

$\varphi, \psi : I \to \mathbb{R}$, found the general solutions of equation (5) in 1999. The next step was done jointly with Páles. They proved that the same solutions can be obtained under one time continuous differentiability. The second important result was the formulation and proof of an extension theorem, which was a joint result with Maksa in 2000. The final breakthrough was obtained with Páles in 2002. This was possible due to the fact that the equation (5) has an implicit monotonicity property. By this property of the unknown functions, one can apply Lebesgue's famous theorem, which states that monotone functions defined on an interval are almost everywhere differentiable. From this, using nontrivial methods, it follows that on some nonvoid subinterval $J \subseteq I$ *the solutions are differentiable* with nonvanishing derivatives. The last step was to show the *continuity* of the derivatives on some subinterval of J. The continuity turned out to be the consequence of the Baire Category Theorem, the properties of the functions of Baire class 1, and a functional equation derived from (5) for the derivatives of the unknown functions. Finally, the following result has been obtained.

Theorem. *The strictly monotone, continuous functions φ and ψ satisfy the functional equation* (5) *if and only if*

(i) *either there exist constants p, a, b, c, d with $acp \neq 0$ such that*

$$\varphi(x) = ae^{px} + b, \qquad \psi(x) = ce^{-px} + d;$$

(ii) *or there exist non-zero constants a, c and constants b, d such that*

$$\varphi(x) = ax + b, \qquad \psi(x) = cx + d$$

holds for each x in I.

This result and the related problems have been extended and generalized in several papers by Zoltán and by his students Burai, Dăscal and by his coauthors Laczkovich, Lajkó, Maksa, and also by J. Jarczyk and Matkowski.

We could continue to present further items of his important, interesting and inspiring results and works, but now we conclude this laudation with a personal remark: We consider him as our master and our friend. We wish you, Zoltán, a happy birthday.