Preparation for and teaching of the concept of area[1]

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In this study the preparation and the teaching of the concept of area as well as the introduction of area standard units and problems related to the computation of area are presented. An experiment conducted in class 5 and 6 is shown, in which we focused on various activities, patterns of work and the application of area computation in everyday life. The efficiency of the experiment was measured on the basis of pre-tests and post-tests.

Introduction

The teaching of this topic offers several options for differentiation and cooperative activities. The use of various activities helps learners in the solution of real life problem situations and makes them more cooperative. Their success in shared work is also motivating for the solution of further tasks. A great deal of attention has been paid to the role of activity and motivation in education by our predecessors. First we would like to refer to Farkas Bolyai, the father and the teacher of the greatest Hungarian mathematician János Bolyai, who formulated several educational principles at the beginning of the 19th century, which are still valid these days. One of Farkas Bolyai's pedagogical principles is that teaching should not be started with suffering, but rather with autonomous activity based on the personality of the child; this activity is to be assisted also later on during systematic learning.

> "… regarding the drive to learn the best thing would be to win the learner's affection for the thing itself, the desire for knowledge is an inherent wish in the soul, and it only has been stimulated." (Farkas Bolyai, 1830)

In the 21st century schools has to cope with new role. Learners have to be provided not only with basic knowledge and skills but also with thinking and communication skills at high level and sociable behaviour.

> "Unfortunately we came to believe that we learn most when we are taught. But the thing is that we learn most when we are motivated and the conditions necessary for learning are provided."(Spencer Kagan, 1994)

In the teaching of the topic the Bruner's representational levels can be observed, but the transition from one level to the other does not go smoothly. The symbolic level is reached too soon and putting down things with figures and letters seems to be a kind of magic trick for many learners. In the formation of the concept of area only few presentations and activities are concerned with measuring large areas outside the classroom and probably this can be the reason why learners are not able to compare and estimate large areas. Activities involving movements in a large area can be motivating for the learners and could contribute to a better understanding of the role of mathematics in science and everyday life. This is also a key issue in realistic mathematics teaching. It is an essential thing that learning should be more than the acquisition of facts. Learners should be able to construct and create their knowledge in an active way. In building up this knowledge children can help each other and shared activities and interactions are highly relevant.

[1] Herendiné-Kónya, E. & Tarcsi M. (2010). Preparation for and teaching of the concept of area. In B. Maj, E. Swoboda & K. Tatsis (Eds.), *Motivation via Natural Differentiation in Mathematics*, (pp. 141-151). Rzeszów: Wydawnictwo Uniwersytetu Rzeszowskiego.

The hypotheses of the research

We assume that the preparation and the formation of the concept of area can be assisted by the following:

H1. Tiling various polygons with various patterns by means of fine movements and walking in the school-yard

H2. Comparing, estimating and determining the area of polygons by geometric transformations and rearrangement

H3. Presenting the use of the concept of area in everyday life and solving practical problems.

The place and time of the research

The developmental sessions were held in the Calvinist Primary School and Secondary school in Kisvárda.
Children filled in the pre-test containing five tasks on 3rd October 2008.
The developmental sessions were held in the afternoon by Mónika Dancs, a student of the teacher training college.
In the experiment two classes participated, class 5/b with 18 pupils and class 6/b with 15 pupils. 25-30 minute sessions were planned.
The post –test was filled by the children on 12th December 2008.

The methodology of the research

The three stages of the research:
- Examination of the level of knowledge regarding the measurement and computation of perimeter and area in class 5 and 6. Two of the tasks will be evaluated together with the post-test. In both classes the knowledge gained in the previous school-year was taken into account, as geometry was not in the curriculum of the given term prior to the survey.
- Seven developmental sessions were held in class 5 and 6. The lessons were taped and photographs were also taken.
- The efficiency of the development was checked by means of a post–test.

Developmental sessions

The aim of session 1
- To make children realize that various shapes can have the same area.
- To demonstrate the relation of size in different areas by means of rotation or cutting and rearrangement.
- To make children realize that the area of plane figures of identical perimeter is not necessarily the same.

The solution of one of the problems is summed up as follows: Out of six various plane figures children were asked to select the ones of identical area. (*Fig.1*)

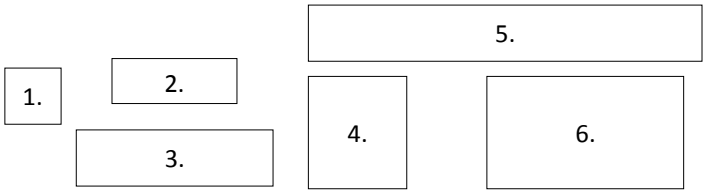

Figure 1

Polygon number 3 and number 5 can be transformed into number 4 and 6 respectively. The solution of the problem is illustrated by an excerpt of the lesson which was taped in class 6.

Teacher:	Is there a polygon of identical area with rectangle number 6 on the blackboard?
Cs.L.:	Well that thin stripe (He meant rectangle number 3.)
M.V.:	It's no way the same!
B.Cs.:	It is only the same size if it covers the whole lot.
Teacher:	Try and find a polygon of the same area out of the ones here.
B.Cs.:	There is no polygon of the same area; some of the smart ones should come here! (He meant the congruence.)
V.L.:	But this is not of the same area!
M.F.:	There are no identical ones!
G.O.:	But there is!
G.O.:	This is of the same area! (He rearranged polygon number 3 into number 4.)
B.K.:	Sure, it won't be OK.
G.O.:	That's it, I told you it's OK. (He demonstrated that is of the same area.)

The aim of session 2

- To make children realize that all of the given polygons can be rearranged into rectangle.
- Sharing each other's ideas while working in pairs.

In one of the tasks of the session children rearranged rhombus, right-angled trapezoid, deltoid, symmetrical trapezoid and hexagon into rectangle while working in pairs. The pairs were given 3 polygons of the five types respectively in order that they could make use of the experience of cutting in the wrong way. Children started rearrangement very creatively and some of them did it with one cut, whereas some of them cut the polygon into small pieces.

In both classes the rearrangement of the right-angled trapezoid was the most demanding. Almost every one noticed that one triangle ought to be cut and placed so that it would be rectangle, but the trouble was that they could not find the proper place of the cut.

In class 5 the rearrangement of the deltoid was also rather challenging, as they were not familiar with the characteristics of deltoid, which is why the majority of them cut it into several pieces. Pupils in class 6 however figured out what the right solution was. (*Fig.2*)

The rearrangement of symmetrical trapezoid (*Fig.3*) and hexagon seemed to be easy for the children.

Some of them who were not able to rearrange the given polygon had the idea to cut the missing parts from an identical leaflet.

Some of the children did not really pay attention to continuous tiling (rearrangement of the rhombus: *Fig.4*)

Figure 2

Figure 3

Figure 4

Children were happy to do these tasks of rearrangement, which is shown by the fact that they did not want to stop the sessions even after 45 minutes. They also came up with new ideas.

The aim of session 3

– To highlight problems related to the continuous tiling of single layer when a rectangle of a given area was tiled with leaves.
– To encourage cooperative activity in pair work.

First they tried to use leaves of almost the same area (*Fig.5*) then they covered the leaflets with leaves of various type and size. (*Fig.6*)

Figure 5

Figure 6

During the activity of covering we observed that boys mostly selected smaller leaves than girls.

As continuous covering was difficult to carry out with leaves, they cut them into various shapes, they preferred rectangle. (*Fig.7*)

Children realised that smaller leaves are necessary to cover the plane figures than bigger one, which is why they used bigger leaves so that they should not glue a lot. As it can be seen in the picture they were rather keen on continuous tiling, the single layer was not important for them. (*Fig.8*)

Figure 7

Figure 8

The aim of session 4

– To make them realize that area of plane figures of the same perimeter is not necessarily the same.
– Using strings of given length forming various polygons and computing their area in the school-yard.
– Group work contributed to the co-operation of learners.

In the school-yard children worked in groups of six. The groups formed various polygons by using string of a given length (*9.6 m*). The square that can be made from the string of *9.6 m* could be covered exactly with the newspapers provided. At the vertex of the polygon one child was standing and another child measured the sides of the polygons, and another one covered the area by newspapers as a given unit.

Class 5

– Only one group was able to determine the perimeter of the polygons they formed and to tile the square formed by them.
– The other group managed to compute the perimeter of the polygons, but they were not able to cover them exactly.
– The third group was not able to complete the task, as they were engaged in watching and following the activity of the other groups.

Class 6

– The group of girls made a square first, then after determining the perimeter they were also smart to cover the area with newspapers cut into rectangles. (*Fig.9*)
– The boys first stretched the string to make an isosceles triangle. While covering they folded the newspaper where it was necessary. During this session we observed that they made an effort to cover the area continuously. (*Fig.10*)
– The girls formed the rectangle in a way that the feet of two children were the vertices, as they were standing astride at a short distance. Having seen this, the boys were more practical-minded and they were standing stride so that the stride should equal exactly the length of the newspaper. (*Fig.11*)

Figure 9

Figure 10

Figure 11

Pupils in class 6 were more dedicated to doing the tasks and they came up with several ideas, which was probably due to the difference between the age groups.

The aim of session 5

– The simultaneous use of the concept of perimeter and area.
– The comparison of the area of polygons by means rearrangement.
– Determination of the perimeter of polygons by measuring.

During the pair-work activity worksheets were used to facilitate the task.

In one of the tasks children determined the perimeter of four polygons and compared their area. (*Fig.12*)

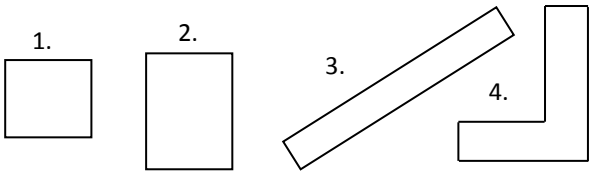

Figure 12

Children measured the sides of the polygons and their calculation of the perimeter was correct.

They soon realized that the area of number 1 and number 3 (*Fig. 13*) furthermore number 2 and number 4 (*Fig. 14*) are identical. They demonstrated it by rearrangement.

Some of them also noticed that despite the fact that the area of rectangle 1 and 3 is identical, but their perimeter is different.

Figure 13

Figure 14

The aim of session 6

– The preparation of the formula of the area of the square and the rectangle.
– Drawing plane figures of identical area on grid.

During the activity worksheets were used to facilitate the task.

They drew polygons by using right-angles triangles of 16 area units so that their area could be the same.

Children actually enjoyed drawing polygons of given area, some of their interesting ideas can be seen in the figures. In *Fig. 15* a solution from class 5 and in *Fig. 16* and *17* the more advanced level of class 6 can be seen.

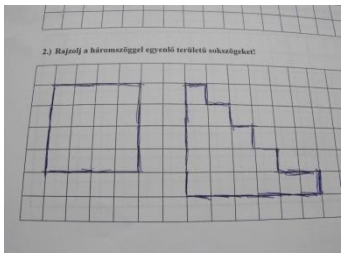

Figure 15

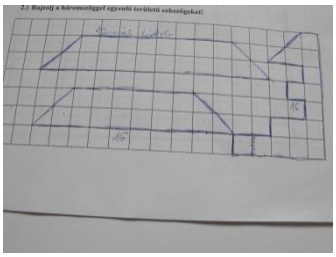

Figure 16

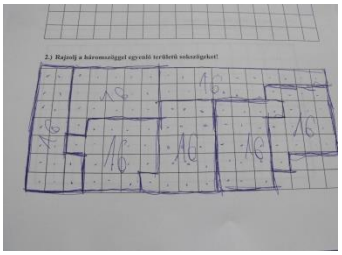

Figure 17

The aim of session 7

– Introduction to the standard units of area.
– Definition of the area of rectangular and square by formula.
– The application of the concept and the computation in real life problems.

Children covered their own desk with sheets of *1 dm*^{2} area. Having gained experience in drawing on grid, the children were aware of the fact that it is not necessary to cover to whole desk in order to compute the area. This is why they placed leaflets cut into *1 dm*^{2} area to the

short and long side of the desk (*Fig.18*). In this way children were able to calculate the area. In both classes there were some children who placed sheets of *1 dm*^{2} round the desk; however this was not really conducive to the formation of either the perimeter or the area. (*Fig.19*)

Figure 18

Figure 19

They did another task in the same way when they covered the sheet of *1 dm*^{2} with sheets of *1 cm*^{2}. Children were rather impatient in placing the *1 cm*^{2} sheets so they did not realize how many small squares are needed to cover the sheet of *1 dm*^{2}. In our opinion this kind of tiling could be useful in the more exact estimation with *cm*^{2}.

Determining the area by means of a formula was rather demanding for pupils in class 5. In order to find the solution they quite often had to rely on counting the squares on grids. Pupils in class 6 reached the level of abstraction where describing the area by letters was easy for them.

In the second part of the session children solved tasks with text in order to examine to what extent they were able to put their knowledge of the concept of area in practice. Children did two tasks together, and they tried to do two tasks on their own. However this proved to be too demanding for pupils in class 5. This failure can be due to their poor comprehension skills.

In the task below two data were provided for one side respectively. *The side of a square room is 30 footprints. One footprint equals 25 cm. How many cm*^{2} *is the area of the room?* As it turned out from the discussion the concept of area has been established, as they were able to determine correctly the area in simple tasks, but in this particular task the too large index-number of the side or the multiplication made them think that they do not need to calculate further on, and they completed the task and they were happy with finding out the length of the side.

Post-test

In the post test we examined

- the establishment of the concept of area during the developmental sessions,
- the reliable knowledge of the concept of perimeter and area, and
- the use of the concept of area in realistic problems.

The post-test, which was filled in by the children on 12th December 2008, contained similar tasks as the pre-test. They took much more delight in filling in the post-test than the pre-test. Two tasks of the pre-test and the post-test are compared and evaluated below. In one of the tasks the given rectangle was tiled with given patterns (*Fig.20*). In the pre-test rotation was needed for tiling with pattern *b*, and in the post-test cutting and rearrangement of pattern *b* was necessary to cover the polygon.

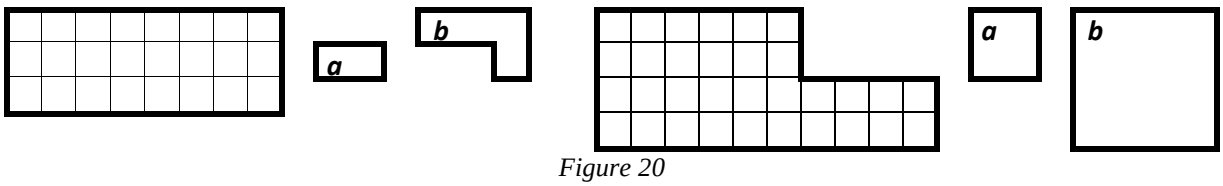

Figure 20

Children in class 5 were able to tile with squares without mistake in the post-test, but fewer of them were able to carry out the rearrangement. It might be due to the fact that the more time should have been spent with rearrangement in class 5. One of the pupils in the pre-test counted the squares in the polygons and calculated the number of tiles by division.

Pupils in class 6 improved a lot. Apparently the activities during the sessions contributed to the clarification of the concepts to a considerable degree. (*Fig.21*)

In the other task we examined how children were able to apply the concept of area in everyday life. The task in the post-test was more demanding than the one in the pre-test, as it included several questions. In this case they calculated not only area but also perimeter, which they might have mixed up.

The task of the pre-test: *The width of a garden is 18 m, its length is 42 m and the buildings are situated on140 m^2. How much is left for other purposes?*

The task of the post-test: *The width of a rectangular plot is 25 m; its length is 34 m. The area of fruit trees covers 320 m^2. How much is left for construction? How many meters of wires are necessary for the fence of the area, if 4 meters are left for the gates?*

In *Fig.21* and *Fig. 22* the percentage of those is shown who completed both parts of the tasks without mistake.

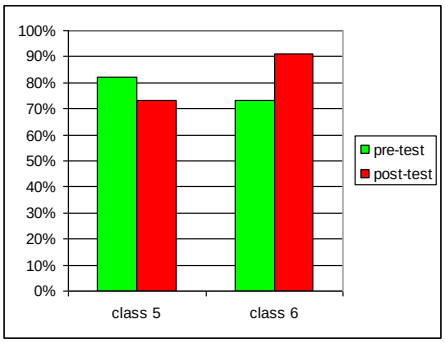

Figure 21

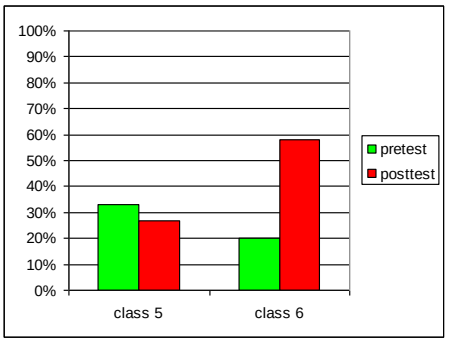

Figure 22

This task of the pre-test was not done by the majority of the pupils in class 6; some of them did not even start to solve it. In class 5 everyone except two pupils attempted to solve it, but they calculated perimeter instead of area most of the time.

In the pre-test very few children drew, whereas the post-test there was hardly anyone, who did not.

In class 5 the skill to calculate area seemingly weakened, which may be due to the poor level of comprehension. When it was only the area that they had to calculate in a task, they managed to do it very well, but the simultaneous application of the two concepts was confusing for them.

The majority of pupils in class 6 took the real life content of the problem into consideration, but the calculation of the perimeter was too demanding for them. During the development sessions we did not really paid much attention to it.

When doing the task pupils in class 5 were concerned only with figures and counting, they rarely discovered logical relationships.

Conclusion

We consider the developmental sessions to be efficient.

H.1 Considerable progress can be observed in class 6. They were enthusiastic from the beginning which can also be seen in their results. Pupils in class 5 were also enthusiastic at the beginning, but during the outdoor sessions in the school yard they were much more interested in games than in learning. Pupils in class 6 very often remembered school-yard sessions, where they moved a lot and gained experience in this way. Progress in the exact tiling can be mostly observed in class 6.

H.2 Children were happy to rearrange and cover the plane figures. Children in class 5 rushed to get their kit containing the scissors, glue and ruler when they were told to cut leaflets or newspapers into pieces. They enjoyed using these things, they liked the figures on them and they tried cut so that these figures would not be damaged. They made a progress in comparing areas, whereas they lagged behind in estimation.

H.3 According to the evaluation of the post-test it can be seen that children are still not able to perceive the size of real life areas and the relationships between them. This is also shown by the fact the in the task above several children distracted the area of the garden from the area of fruit trees.

After analysing the developmental sessions and the tests we came to the following conclusions. In the future more attention should be paid to the following:

- In class 5 the difference between the concept of perimeter and area should be highlighted by means of more developmental activity.
- Children should measure more both small and large objects in order that their estimations get near reality.
- More co-operative activities are needed in the upper primary, because these engaging activities can arouse children's interest, and the shared activities can facilitate the understanding of complicated relationships.

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