AREA MEASUREMENT TEACHING IN A GRADE 6 CLASSROOM

Eszter Herendiné-Kónya

University of Debrecen, Hungary

In our paper we report on one part of a multi-stage teaching experiment conducted in 6th Grade, in which we dealt with the comparison of the areas of figures by visual estimation first, than by superimposing one onto the other, furthermore with the area measurement using a square grid. Our main conception is to apply general principles of measurement teaching for the area measurement.

INTRODUCTION

Examining the performance of Hungarian students of Grades 5-12 in connection with area measurement, we found many deficiencies and thinking failures (Herendiné-Kónya, 2014, 2015b). Students are often capable to solve only tasks which require the use of simple formulas, and we experienced the nonsense use of the learnt formulas. When determining for example the area of irregular 2-dimensional figures they are not able to use the additive property of the area, they do not see the parts in a compound figure, which area could be easily calculated. They have little knowledge and experience about the conservation of the area, that is, not only congruent shapes have the same area. In the light of this background, it seems reasonable to identify those teaching movements that trigger the explored problems and to design a teaching experiment that tries to avoid and exclude them.

In our paper we report on one part of a multi-stage teaching experiment conducted in 6th Grade, in which we dealt with the comparison of the areas of figures by visual estimation first, than by superimposing one onto the other, and with the area measurement using a square grid. The aim of this research is to analyse the teaching process, i.e. the teaching methods and cognitive characteristics of students.

TEORETHICAL FRAMEWORK

Research often reveals poor understanding of the processes used for area measurement of plane figures. Though it is generally accepted that mathematics should be taught through understanding but in the topic of area it would seem that children often rely on the use of formulae with little understanding of the mathematical concepts involved. Some studies point out that in order to determine the area, the measurement should be done indirectly: firstly measuring the lengths, secondly calculating the area (Nitabach & Lehrer, 1996; Tsamir, 2003; Murphy, 2010). In the initial phases of the concept formation it can be useful to consider area as a quantity independent from length. The study of Curry, Mitchelmore and Outhred (2006) analyses the measuring of length, area and volume independent from each other according to five measurement principles:

> … 1. the need for repeated units that do not change; 2. the appropriateness of a selected unit; 3. the need for the same unit to be used to compare two or more objects; 4. the relationship between the size of the unit and the number required to measure; 5. the structure of the repeated units. (Board of Studies NSW, as cited in Curry et al., 2006, p. 377).

Baturo and Nason (1996) describe the gist of measurement as continuous quantities being divided into equal discrete units and then counted. The measuring can be done in two ways: we take a unit and cover the whole quantity successively, or we take the required size unit and cover the whole quantity at once. Taking into consideration the nature of area measurement, only the latter approach can be used. The appropriate measurement tool is the transparent grid, especially the square grid. However we have to take into consideration that the investigation of different tessellation patterns using congruent tiles should precede the introduction of the grid as a measurement tool. In order to accelerate the process of tessellation the students are able to construct the grid from many square tiles spontaneously (Herendiné-Kónya, 2015a). What's more, Kamii and Kysh (2006) draw attention of the risk of counting squares in the process of area concept formation. Their experiment showed that "… for the 94% of the investigated students in Grade 8, squares were rigidly inviolable, discrete objects rather than objects that could be used as units that covered an area." (Kamii & Kish, 2006, p. 113).

The formulas of area calculation are introduced too early, long before a stable *concept image* (Tall & Vinner, 1981) would be formed in the students' minds. If the calculation rules are not linked to actual experiences, the knowledge of area is not effective (A. Baturo and R. Nason, 1996).

In understanding area measurement, area conservation has a crucial role, that is, the fact that the area of a figure doesn't change if the figure is cut up and a new figure is composed from the parts (Piaget, Inhelder & Seminska, 1960). According to Kordaki (2003), area conservation, area measurement and area formulas should be taught in an integrated way, in order to develop all three aspects. The study also shows that the type of the figures may have a role in recognising the area conservation.

Kospentaris, Spyrou and Lappas (2011) claimed that recognising area conservation could cause problems for secondary school students and even for first year university students. The idea that only the areas of congruent figures are equal is very strongly rooted. They investigated the positive and negative features of the visualization and discussed the role of the visualization in the area conservation and indirect comparison.

In Zacharos' paper (2006) we can read about the teaching practice of area measurement and the mistakes of concept formation. He saw the problem in the too early introduction of formulas, but also referred to the misunderstandings which roots from the wording. Unlike the word 'length', the 'area' is not used in the same way in our everyday life as in mathematics. Area is not only used to denote a measure, the quantity describing a plane figure, but very often by the word 'area' we mean the domain itself, and it can also occur that it means the multiplication of the width and length as it could be linked to the rectangular figure.

The cited studies confirm that for the formation of the area concept, measuring practice is needed independently from the length. So we applied the general steps of teaching measurement: the direct comparison of quantities without measuring; the need for repeated (standard or not standard) units; estimation; the relationship between the size of the unit and the number required to measure (reciprocity); choosing the appropriate unit for a concrete quantity. (Herendiné-Kónya, 2013).

RESEARCH QUESTION AND METHODOLOGY

The focus of our recent research is on the activities related to area estimation and conservation, furthermore area measurement using grid.

Research question

1. How is possible to realise certain activities related to the comparison, estimation and direct measurement of areas in a regular classroom environment?

2. What kind of typical mistakes make the students as well as the teacher in the observed teaching/learning process?

A teaching experiment was carried out among a group of 6th grade Hungarian students in December 2015. According to the discussion with the class teacher before the experimental teaching we considered the mathematical knowledge and skills of these students as average or a slightly below average. By the time, they have already learnt the methods of calculating the perimeter and area of rectangles and squares as well as the use of a few standard units like cm^2 and m^2. However the students haven't learnt the conversion between them and the area formulas connected to quadrilaterals or triangles. The teaching experiment was based on the curriculum framework topic *Measurement*. We designed the activities for regular classroom situation and paid special attention to being in line with the curriculum that is we kept to the required teaching time of the topic and didn't plan any extra lessons. In our opinion in this way the experimental activities could turn into an integral part of the teaching practice.

Four task were designed with different duration in time and were introduced in two consecutive 45 minutes classes. The experimental teaching material and the teaching aids were compiled by the author and the lessons were conducted by the class teacher in accordance with our guidelines. The tasks were done in groups, whereas setting the tasks and the discussion of the experience took place in the whole class. We applied cooperative teaching method, because it is suitable for these activity-based lessons: students have the opportunity to try, explain and control their ideas in a small group of their classmates. They were familiar with this way of learning; their teacher was expert in organization of students into groups and in supervision of the classroom work. The children divided themselves into 5 groups, one in which there were 2 members and in the rest there were 4. All the lessons were voice-recorded, notes and photos were taken.

DISCUSSION AND RESULT

On the first class we completed the first three tasks. On the second class we dealt with the fourth task then the teacher continued the teaching prescribed in her syllabus in a traditional way.

Task 1 – Recognizing that different shapes could have the same area by visual estimation at first and by equidecomposition later

We showed 8 figures with different shapes on the interactive whiteboard (Figure 1), and asked students to compare and order its area using visual estimation. The suggestions of the five groups were written on the blackboard.

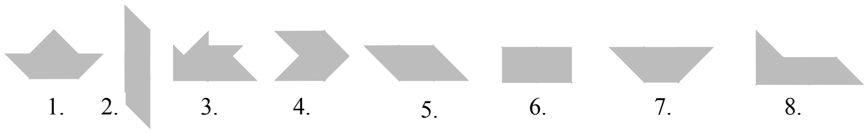


Figure 1. Figures which areas are equal.

As a next step we divided a square on the interactive whiteboard into 4 congruent triangles. The students came to the board and covered the figures using ‘drag and drop’ technique. After two attempts somebody already recognized that all of the figures can be covered with 4 triangles, but the classmates wanted to try the technological tool to control this statement (Figure 2). The interactive whiteboard seemed to be good tool for such kind of tangram-like activity, but we found that the emphasis was mainly on the use of technology and not on the essence of the activity itself. Nevertheless the students came up to the conclusion, that “It doesn’t matter that the figures have different shapes, their areas are equal.”

A class discussion was followed about the estimations of the order of the areas. It was clearly seen that the rectangle was at the beginning while the figures number 3 and 8 were at the end of the line.

1	T:	Why did you think that the figures number 8 and 3 are the largest in area?
2	S1:	Because they seem to be large.

3 S2: … because they have such interesting shape and we didn't know how large could they be.

The students listed several other reasons like "it's composed from many parts", "it has strange shape", "it hasn't a regular shape", "it's sharp", "it has more than 4 corners".

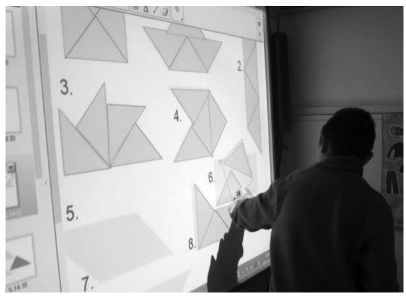
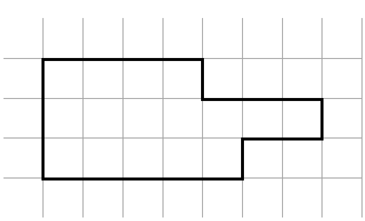


Figure 2. Covering the figures with congruent triangle.

Task 2 - Constructing different shapes of the same area on a square grid

A 4×4 square was given on a grid. Students had to draw 3 figures with different shapes but the same area as the square had. Some additional questions of pupils helped to understand the problem: "Should it be constructing from triangles?", "Is it allowed to use *half-cube*?" The students designed the figures together thereafter one of them explained their solutions to the whole class. Each group drew correct figures. All but one figure were rectilinear. We conclude from the way of students' justification that they are able to cut the figures into rectangles in their mind that is to say they applied the additive feature of the area unconsciously. For instance (Figure 3): "Here are … 4 times 4 … ehm…. 4 times 3 then a double vertically and the other horizontally." The multiplication rule according to the rectangle also came up (Figure 4): "Well, we made this long rectangle, here are 8 squares and here are 2." Only one group thought on the half-square and the right angled triangle as possible component.



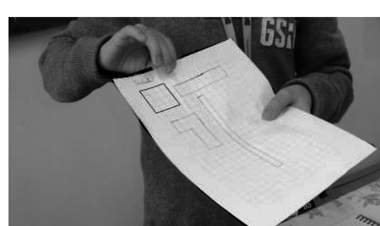


Figure 3. Rectilinear figure of area 16 □. Figure 4. Explanation of the constructions.

Task 3 – Comparing the areas of different shapes directly; then measuring these quantities using a centimetre grid as an instrument for measuring area

The figures were cut out from coloured paper and their areas, except of one, were whole numbers measured in the given unit squares (Figure 5).

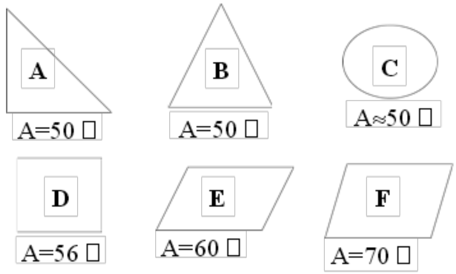


Figure 5. Six figures cut out from coloured paper.

The direct comparison of the areas of some pairs were easy (e.g. F>D), but some of them requires mental decomposition and visual comparison (e.g. E>B). One of the important experience gained from this activity is that most of the students didn't think that such kind of shapes like C has any area: "Does the *circle* have any area at all?!" or "We don't think so, … we don't believe that the *circle* has an area." One of the groups ignored the figure C when ordered them from the smallest to the largest area. When they started to measure the areas using grid, we observed the trouble with the ellipse again:

1 T: Put the grid on the figure and count how many squares convert it.

2 S': And what should we do with the *circle*?

3 T: Figure out it approximately!

Table 1 shows the results of the measurement using a grid. We indicate the correct measure of the areas in the table's last row. Analysing the solutions of the groups it can be seen that

- the area of the rectangle (Figure D) is almost correct.
- the areas of the triangles and parallelograms are measured by counting the squares directly instead of converting to rectangles. They got for instance 48 or 59 instead of 50 or 60.
- the order of the areas after measuring was essentially correct in every case.
- the areas of every triangles and almost of every parallelograms were considered as smaller in area than the correct value.

	Figure A	Figure B	Figure C	Figure D	Figure E	Figure F
Group 1	49	50	53	56	59	69
Group 2	42	47	51	56	59	70
Group 3	48	48	42	56	55	60
Group 4	49	48		64	68	68
Group 5	45	43	43	56	53	64
Correct	50	50	≈50	56	60	70

Table 1: The values of the areas measured in unit squares.

We asked students how they counted the squares in the case of part-squares around the boundary. One of the answers: "I count in a way that I numerate first the whole squares after that the halves and if there are two halves, I count it as a whole. … and the *smaller half* with the larger one take together a whole." (She uses the word 'half' as synonym of the word 'part'). We recognised that all of the students thought in this way: mentally tried to combine part-squares to make whole squares.

Task 4 – Estimating relative areas of two irregular shapes after that measuring them using grid

By planning this task we took into consideration that the existence of the area of a shape with irregular outline caused trouble for many students. Our purpose was to give them further practice in the use of centimetre grid for measuring area, and in estimating relative areas. The students had to compare first, and then measure the area of two Hungarian counties on the map. (Figure 6). After that they searched the areas in km^2 on the internet and controlled their relative estimation.

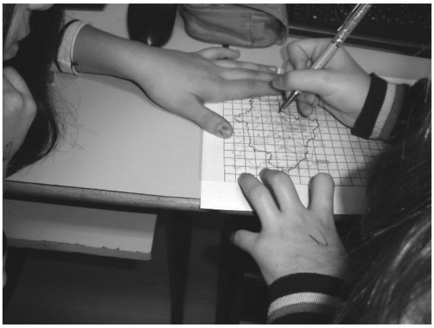


Figure 6: Practice in the use of the grid.

The results gained by counting the whole- and part-squares shown big differences, so a teacher-led discussion was initiated how to improve the accuracy in area measurement.

1	T:	How could we measure more precisely, ….., how could we determine even these small overhanging parts?
2	S1:	No way! It's not possible!
3	S2:	It's not possible with this (*he shows the grid*).
….		
4	S3:	Smaller grid!

After a relatively long time S3 gave the answer the teacher waited for, but the idea of the need for smaller measurement unit wasn't well elaborated in the class this time. (We should be noted that the teacher's question wasn't well thought-out.)

Traditional way of teaching

After the Task 4 the class teacher continued the 45 minutes class in a traditional way using the material prescribed in her syllabus. From this part of the lesson we pointed out two episodes: 1) making connection between the centimetre grid and the cm^2 as a standard unit; 2) calculation of the area of a rectangle by measuring the lengths of the sides.

1) After students established the size (1 cm) of one square on the grid the following discussion was detected:

1	T:	Draw a square, and denote its sides by *a*. We know that the length of the side *a* is 1 cm. How can we calculate the area of a square?
2	S':	*a* times *a* (*together*)
3	T:	Why is it possible that the area of a square is *a* times *a*?
4	S1:	Well, … because of the same size.
5	T:	The same size, good … What is the area of this square?
6	S1:	2.
7:	S2:	1 times 1.
8:	T:	And how many cm^2 is this? *(she ignores the wrong answer)*
9	S2:	1.
10	T:	1 cm^2.

The teacher emphasised the symbol *a* which often used in area calculation tasks and determined the area of the square applying the formula $a \times a$ and not the concept of cm^2 itself. The lines 3-5 show that the student S1 as well as the teacher thought only on the particular method and not on the concept of area in general.

2) The discussion below illustrates the conflict between the experimental and the traditional way of teaching:

1	T:	Measure width and length of your booklet using ruler, and then calculate the area. How can we calculate the area of a rectangle?
2	S1:	*a* times *b*

3 T: And why is it *a* times *b*? Why isn't it else?

4 S1: Because the pieces haven't the same size …

5 T: Very well, because the two sides haven't the same size … ehm … and the areas are these small squares (*she shows on the board*)

It's clear that the teacher didn't have the intention of connecting the formula and the concept of area but she wanted only to choose the appropriate formula again ($a \times a$ or $a \times b$). Furthermore we can recognise the duality in the teacher's explanation as it is seen in line 5. It seemed that she understood the essential parts of the experimental activities and taught them, but she returned back very quickly to her traditional methods and tasks. So we can say the she actually 'put into brackets' i.e. neglected the experimental teaching material and confirmed 'the area is measured by ruler because it is length multiplied by width'.

CONCLUSION

Regarding our research question we summarize the experiences gained from the two lessons described above in the following points:

- The fact that a shape with a curved border also has an area was surprising most of the 6th graders.
- The students used the grid as a tool for measuring area in a right way and in the case of the part-squares around the boundary they combine the parts to make a whole squares.
- The statement that the accuracy in area measurement can be improved by using other grid with smaller squares requires more tasks. In this case the need for the introduction of different standard units would be better established.
- However the teacher understood the true concept of area as the number of unit squares which can be combined to cover the given figure, in particular teaching situations she reduced the area concept to a specific calculation method. We found that there are certain habits in teaching which are hard to give up or change.

REFERENCES

Baturo, A. & Nason, R. (1996). Student teachers' subject matter knowledge within the domain of area measurement, *Educational Studies in Mathematics, 31(3)*, 235-268.

Curry, M., Mitchelmore, M, & Outhred, L. (2006). Development of children's understanding of length, area, and volume principles. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education, Vol. 2*, (pp. 377–384), Prague: PME.

Herendiné-Kónya, E. (2013). A méréstanítás általános szempontjai [The general principles of teaching measurement], In: E. Herendiné-Kónya (Ed.), *A matematika* *tanítása az alsó tagozaton* [Teaching mathematics in primary school], (pp. 240-248), Budapest: Nemzedékek Tudása Tankönyvkiadó.

Herendiné-Kónya, E. (2014). How can high school students solve problems based on the concept of area measurement? In: A. Ambrus & É. Vásárhelyi (Eds.), *Proceedings of the 15th ProMath Conference 30 August-1 September 2013,* (pp. 95-107), Eger: Mathematics Teaching and Education Center.

Herendiné-Kónya, E. (2015a). The formation of area concept with the help of manipulative activities, *Teaching Mathematics and Computer Science, 13/1*, 121-139.

Herendiné-Kónya, E. (2015b). The level of understanding geometric measurement. In: K. Krainer, & N. Vondrová (Eds.), *Proceedings of the Ninth Congress of the European Society for Research in Mathematics Education,* (pp. 536-542), Prague: Charles University in Prague, Faculty of Education and ERME.

Kamii, C. & Kysh, J. (2006). The difficulty of „length×with": Is a square the unit of measurement? *Journal of Mathematical Behavior, 25*, 105-115.

Kordaki, M. (2003). The effect of tools of a computer microworld on students' strategies regarding the concept of conservation of area. *Educational Studies in Mathematics, 52(2)*, 177-209.

Kospentaris, G., Spyrou P. & Lappas, D. (2011). Exploring students' strategies in area conservation geometrical tasks, *Educational Studies in Mathematics, 77(1)*, 105-127.

Murphy, C. (2010). The role of subject knowledge in primary student teachers' approaches to teaching the topic of area.In: V. Durand-Guerrier, S. Soury-Lavergne & F. Arzarello (Eds.), *Proceedings of CERME6, January 28th-February 1st 2009*, (pp. 1821-1830). Lyon: Institut National de Recherche Pédagogique.

Nitabach, E. & Lehrer, R. (1996). Developing spatial sense through area measurement. *Teaching Children Mathematics, 2(8)*, 473-476.

Piaget, J., Inhelder B. & Seminska, A. (1960). *The child's conception of geometry*, London: Routledge and Kegan Paul.

Tall D. & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12 (2)*, 151–169.

Tsamir, P. (2003). Using the intuitive rule more A – more B for predicting and analyzing student's responses in geometry, *International Journal of Mathematical Education in Science and Technology, 34(5)*, 639-650.

Zacharos, K. (2006). Prevailing educational practices for area measurement and students' failure in measuring areas, *Journal of Mathematical Behavior 25*, 224-239.