SOLVING OF REAL SITUATIONS BASED PROBLEM – EXPERIENCE WITH TEACHER TRAINING STUDENTS

Gabriella Ambrus – Eszter Kónya

Eötvös Loránd University, University of Debrecen

An important task is for the future teachers to teach their students for the use of their mathematical knowledge even in everyday (real) situations. This task requires often from the students a change of view – according to our tests. Last year we started to work out a program in Hungary which can (may) help them to develop in this subject. Now we show some details of this program and discuss our first experience.

INTRODUCTION

Word problems with real context appear already in the first mathematics book in 1476 (Teviso Arithmetic) (Verschaffel et al, 2010, p. 11). In Hungary there is plenty of word problems connected to the everyday life in old arithmetic books as well, i.e. in the „Arithmetica" of Maróthi (1743). Maróthi emphasised the importance of the real world tasks[1] in the introduction of his book already. One of his goal was to give an overview about the possible applications of the topics discussed in the book, so he chose the tasks from different fields of the everyday life.

In the secondary school mathematics the Wlassics Curriculum (1899) brought a major change in Hungary which included the goal of „understanding the simple numerical relations of practical life". The curriculum emphasized the importance of computing throughout mathematics and included, among others, interest and loan calculations that are part of daily life and the use of trigonometry in surveying tasks. (Beke, 1911, Beke & Mikola, 1909, 1911). In the textbooks written by Emanuel Beke (1862-1946) (Beke was the leader of the Hungarian mathematics teaching reform around 1900), there were lot of word problems based on real life. „Students must realize how many links there are between mathematics and everyday life, sciences and our entire perception of the universe." (Beke & Mikola, 1911, p. 200).

It is important to notice that the mentioned Hungarian reality based tasks are closed i.e. there are only one possible correct solution usually.

If the task „ … requires translations between reality and mathematics what, in short, can be called mathematical modelling. By reality, we mean according to Pollak (1979), the 'rest of the world' outside mathematics including nature, society, everyday life and other scientific disciplines." (Blum & Borromeo Ferri, 2009, p. 45). Lesh and Zawojewski (2007) discussed different instructional approaches concerning problem solving. One of the existing approaches assumes that the required concepts

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[1] We use the term „task" here as an umbrella term that includes the concept of problem, too.

and procedures must be taught first and then practiced through solving routine world problems. Another approach presents students a repertoire of problem solving strategies such as „draw a diagram," „guess and check," „make a table" etc. and provides a range of non-routine problems to which these strategies can be applied. A rich alternative to these approaches is one that treats problem solving as integral to the development of an understanding of any given mathematical concept or process, mathematical modelling is one such approach. (English, & Sriraman, 2010).

Concerning the recent curriculum in Hungary[2] mathematical modelling has to be present in instruction.

So we can underline that the reality based contexts are crucial for engaging students in mathematical modelling and for preparing students to use mathematics beyond the classroom.

THEORETICAL BACKGROUND

In the Hungarian school practice mostly closed problems (or such problems that appears at first sight to be closed) are used, (Ambrus, 2004) despite of the fact, that the open problems are highly important in the teaching of mathematics (Pehkonen, 1995; Munroe, 2015). The method „Open approach" – the use of open ended tasks on the mathematics lessons – was worked out in Japan in the 70-s of the last century. At the same time became popular the so called „tasks for researches and investigations" in England (Silver, 1995).

In a closed task the starting and goal situation is exactly given. In the open tasks the starting, the goal or eventually both situations aren't exactly given[3]. If the starting or the goal situation is not exactly given) „students may end with different but equally right solutions, depending on their additional selections and emphasis done during their solution processes." Pehkonen, 1999, p. 57) The types of open problems Pehkonen (1999) categorized as follows: (1) Investigations (the starting point is given); (2) Problem posing; (3) Real-life situations (which have their roots in the everyday life); (4) Projects; (5) Problem fields; (6) Problems without a question; (7) Problem variations („what-if-method").

Most students struggle when faced with complex and ill-structured tasks because the strategies taught in schools and universities simply require finding and applying the correct formulae or strategy to answer well-structured, algorithmic problems (Ogilvie, 2008). They are asked to solve rarely open-ended challenges, but exercises in familiar tasks, with an emphasis on completing these tasks quickly and efficiently (Schoenfeld, 1988). „The usual practice involving routine word problems, ... engages

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[2] National Core Curriculum (Nemzeti Alaptanterv, NAT) 2012.

[3]According to a more general conception, problems with exactly given starting and goal situations can be also considered as open tasks in the case when several possible way of solutions can be formulated for the task (Wiegand & Blum, 1999; Büchter & Leuders, 2005).

students in a one- or two-step process of mapping problem information onto arithmetic quantities and operations." (English & Sriraman, 2010, p. 267). Students tend to exclude real-world knowledge and realistic considerations from their solution processes (Puchalska & Semadeni, 1987; Verschaffel, DeCorte, & Lasure, 1994; Csíkos, Kelemen, & Verschaffel, 2011).

We experienced many times in our teaching practice when the students work with real life situation where the starting situation is not exactly given that they

- can't solve the problem;
- try to make connection between the given numbers and the question neglected the concrete problem situation;
- may not realize the openness and work with the given data;
- complete the text (automatically, „as usual", without considering any assumption) in order to close the task and solve it obviously;
- may end it with (at least one or more) different equally correct solutions depending on additional selections, (there is an expectation to find at least one possible solution of the task connected to a possible initial assumption).

In order to better highlight our approach concerning open world problems we present here an example, the „Pocket money" problem.

OUR PREVIOUS RESEARCH: THE „POCKET MONEY" PROBLEM

The „Pocket money" problem is a text-variation of another task "Dresses of the queen" (Ambrus & Szűcs, 2016). The variant in connection with money seemed to be more appropriate – according to the previous surveys – for students, especially for older than 4th grade students. (Ambrus, 2016).

The „Pocket money" problem
Since Pisti[4] moved to a new house with his family, he has received his pocket money, 1000 Hungarian forints, weekly. He has saved all his pocket money since they moved in. How many days have they spent in their new home if Pisti has saved already 35 000 Hungarian forints? (Ambrus, 2016)

At first the problem seems to be a closed whose solution is 35 times 7 which equals to 245. Nevertheless, it is an open problem as for example neither the date of the arrival of the family nor the days on which the pocket money is given to Pisti are specified in the problem. Moreover, in the problem „a week" can mean a calendar week or seven consecutive days.

Thus, for example if he receives his pocket money every Monday they must have spent 35 Mondays in their new home which means at least 34 whole weeks. Following from this, the solution can be 34x7+1=239 days. On the other hand, they could have spent at most 6 days in their new home before handing Pisti his first

[4] Hungarian boy's first name.

(Monday) pocket money thus the family have spent at most 35x7+6=251 days in their new home. Although the problem requires not much mathematical knowledge, a solution with systematically arranged assumptions and solutions may be a challenge for 14-18 years old students as well (Ambrus, 2016).

There were several investigations with the problem „Pocket Money", between 2012 and 2016. Hungarian upper primary and secondary school students from different types of school and university students worked with the task individually without any help. The students could use as much time as they needed (usually, the solution required no more than 10-12 minutes). The main question was: Did students recognize the openness of the problem i.e. did they consider at least one assumption which were used in the solution? Our hypothesis was, that at upper grades (and at the university) most of the students can realise the openness of the task, because on the one hand the situation is easy to imagine, on the other hand the calculation needed is a routine process, so it remains time to think about the situation. The proportion of the open solutions in different groups is shown on the Figure 1 the number in the brackets shows the number of the students who took part in the investigation).

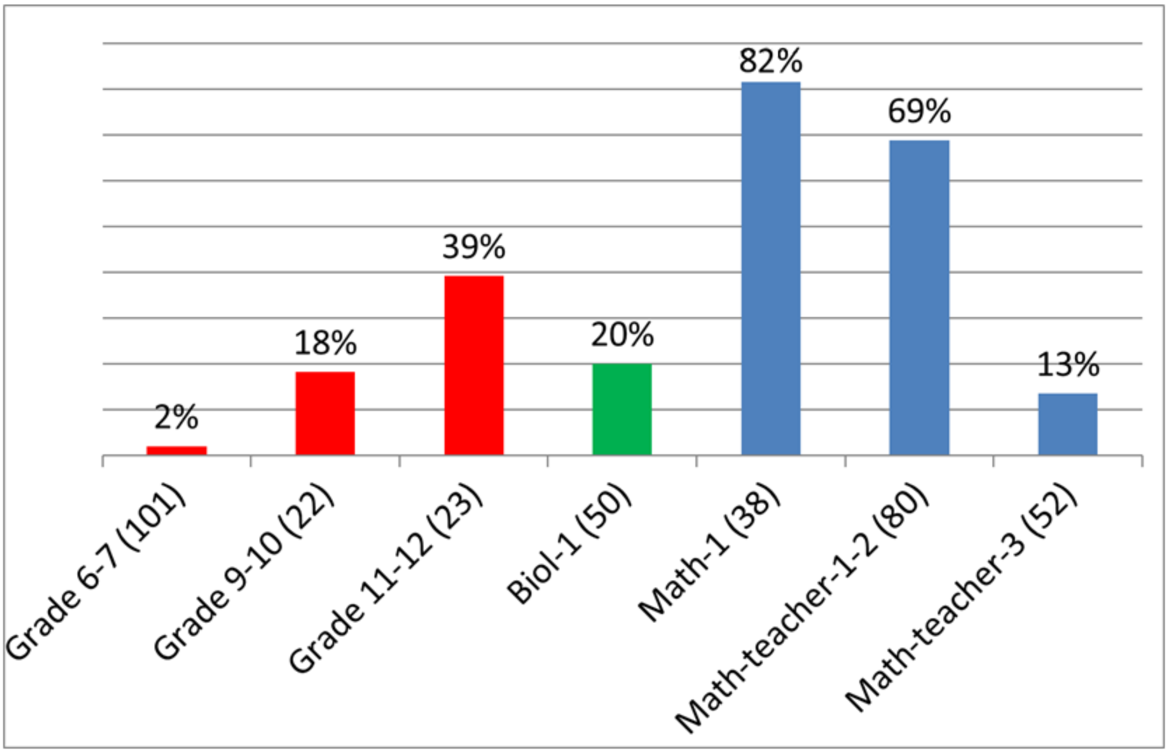

Figure 1: The open solutions of the „Pocket money" task in different groups.

The result shows a diverse picture and led us to some consideration as well. We couldn't identify clear pattern concerning the age or the mathematical interest of the students. It is obvious, that an open solution for the task not appears really often in higher classes and the interest in mathematics seems to be only one of the aspects which may have an impact on the solution. At the same time the previous school exercises and the teacher's belief could be remarkable reasons of this phenomenon. As background for the results can/must be considered (1) in what extent the students had the possibility to solve tasks of this type or (2) their belief about mathematics. Furthermore not less important factors are (3) their teacher's belief (about mathematics) and (4) the way of teaching.

Although the specific relationship between teacher's beliefs and their teaching practice is not known (Maaß, 2011, Thompson, 1992), the teachers' beliefs have a decisive influence on their students' beliefs and what is more, the image about mathematics for the students is largely decided in the school (Grigutsch, Raatz & Törner, 1998). So the belief of the teacher about mathematics (i.e. that he/she has a static or dynamic view about mathematics) influences the way of thinking (schematically or not) of their students by solving word problems.

THE EXPERIMENTAL RESEARCH PROGRAM

With respect to the results of our previous research we concluded that the future mathematics teachers during their training have to solve open problems in real world context and have to deal with the teaching methods of such kind of problems. We elaborated a developmental program for our teacher students in order to develop their awareness towards open real world problems and to gain first experiences for a planned teacher training material concerning our Project[5].

In this paper we discuss about two research questions we formulated concerning the first teaching experiment:

1. Do teacher students recognize whether different problems are open or not?
2. How students think about the open problems?

METHODOLOGY

We elaborated and tested the experimental program in the frame of the so called „Problem solving seminar" course, which is obligatory part of the teacher training curriculum in Hungary. The structure of the „Problem Solving Seminar" let to involve different topics in connection with problem solving practice, so using open, real world problems was absolutely in line with the aim of the course.

We tried our ideas at two universities, at the Eötvös Loránd University in Budapest and at the University of Debrecen. The number of participants in the group in Budapest was 16, while in Debrecen 18. Their main subjects are Mathematics and a

[5] Content Pedagogy Research Program of the Hungarian Academy of Sciences, 2016-2020.

second discipline, like Physics, Informatics, History, English etc. The students participated in our course were first, second or third graders.

During the semester we worked five times (out of twelve) with open reality based problems. First they solved the „Pocket money" problem; afterwards we had a 90 minutes session with (simple) open tasks in reality context. Students worked individually first with the problem, thereafter we discussed the solutions and possible questions. The further sessions took place in the first part of the seminar lessons, where we worked on the similar way with the tasks. On both universities the students worked practically with the same tasks, the seminar lecturer were the authors of this paper.

Schedule of the course

- Week 1-2: „Pocket money" problem – test then 4-5 simple open problems (in real world context) – discussion
- Week 3-6: 2-3 open problems besides closed tasks
- Week 7: „Party" problem – test
- Week 8-11: 2-3 open problems besides closed tasks
- Week 12: „Season ticket" problem – test

Besides the „Pocket money" problem the student had the opportunity to work alone (and write down their own solution) twice a semester. They had to solve an open reality task as well on the two usual seminar tests (among other problems related to the basic material of the subject).

Photos, notes and audio recordings were made during the sessions, in addition to the two written works.

The test problems and their coding

The „Party" problem
Karcsi[6] *has 5 friends and Gyuri*[7] *has 6 friends. Karcsi and Gyuri decide to give a party together. They invite all their friends. All friends are present. How many friends are there at the party?* (Verschaffel et al, 1994)

The task seems to be simple, the expected answer is: 5+6+2=13. Understanding the real situation deeper we realise that we do not know whether Karcsi and Gyuri have common friends or they are friends at all. Depending on the initial assumptions we can give more correct solutions.

The „Season ticket" problem
The monthly public transport pass for students is available with any starting day and valid for 30 days including the starting day. Szilvi[8] *buy such a pass every time. Her*

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[6] Hungarian boy's first name.

[7] Hungarian boy's first name.

[8] Hungarian girl's first name.

first pass was bought in 05.01.2015. She remembers that her pass bought in June was valid until the day she travelled to her grandmother. In which day did travel Szilvi to her grandmother in this summer? (Vancsó & Ambrus, 2007)

At first sight the result is July 2, but some circumstances aren't specified: (1) She bought pass exactly in every 30 days? (2) She skipped some days? (3) It could happen that she had two valid passes on one day?

For the evaluation of student's written answers we used the following coding system (Table 1). This way our results can be compared with other surveys using similar type of tasks and coding.

First code	**Description**	**Examples from the solutions of the „Pocket money" problem**
1	Expected answer which results from the application of the arithmetic operations elicited by the problem statement.	7x35=245
2	Expected answer with a technical mistake.	There is a mistake in the counting.
3	Realistic answer which follows from the effective use of real-world knowledge about the context of the task.	The real situation was at least partly considered in the solution, eg. 245-251 days or with some mistake: e.g. 245-252.
4	other answer	Eg. error resulting from misunderstanding: 35 days.
0	no answer	
Second code	**Description**	**Examples from the solutions of the „Pocket money" problem**
1	Any comments which refers to hesitation concerning the answer.	„245 days, but it can change depending on which day he receives the money" „245, but I'm not sure that this is a correct answer".
0	No such a comment.	

Table 1: Coding system for the student's solutions, on the basis of the work of Verschaffel et al. (1994).

The responses coding by 30, 31, 11, 21, 41 refers to the cases wherein a student gave a *realistic* answer to the problem Verschaffel et al. (1994).

We show some examples in the Figure 2, 3 and 4 in order to better highlight the way of coding.

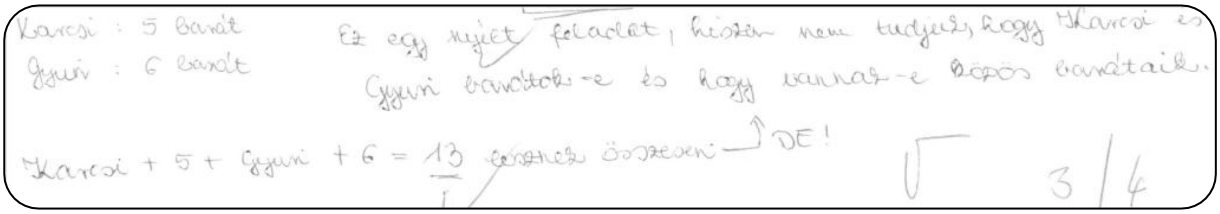


Figure 2: Solution of the „Party" problem coding by 11.[9]

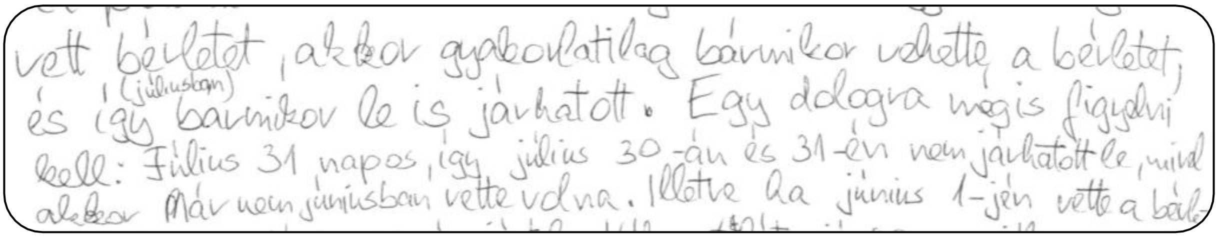


Figure 3: Solution of the "Season ticket" problem coding by 31.[10]

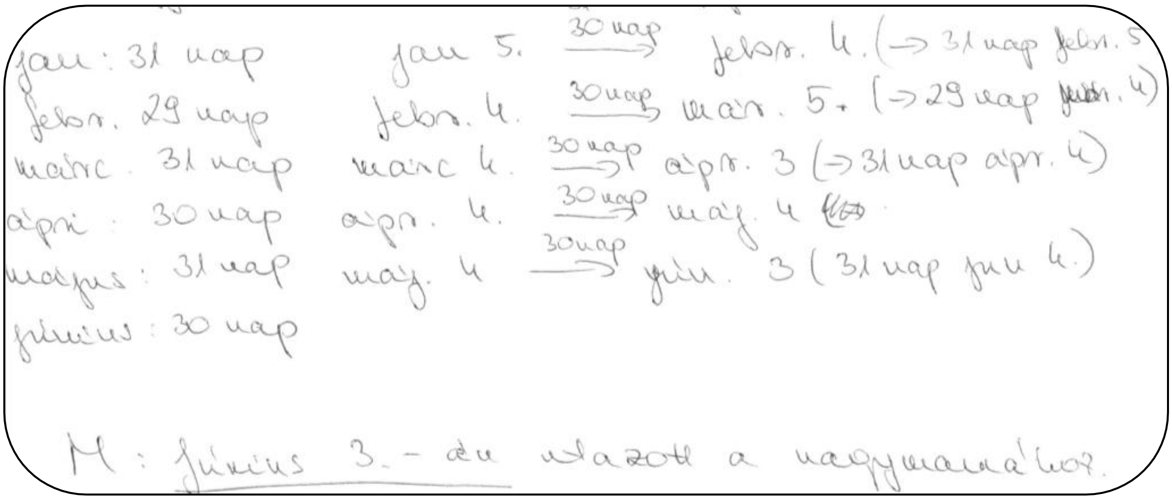


Figure 4: Solution of the „Season ticket" problem coding by 20.[11]

[9] „The total number is Karcsi+5+Gyuri+6=13, BUT! this is an open task, because we don't know whether Karcsi and Gyuri are friends or they have common friend at all."

[10] „She could buy the ticket anytime, so in July it was valid in July anytime, except of 30 and 31 of July, because in this case she didn't buy it in June."

[11] The student calculated the wrong (answer, because he/she didn't considered that the last pass was bought in June) date supposing – but not mentioning it – that Szilvi bought pass exactly in every 30 days.

RESULTS

We investigated the solutions of the three open problems concerning the first research question.

Figure 5 shows the proportion of the open answers (each student produced one answer) in both groups.

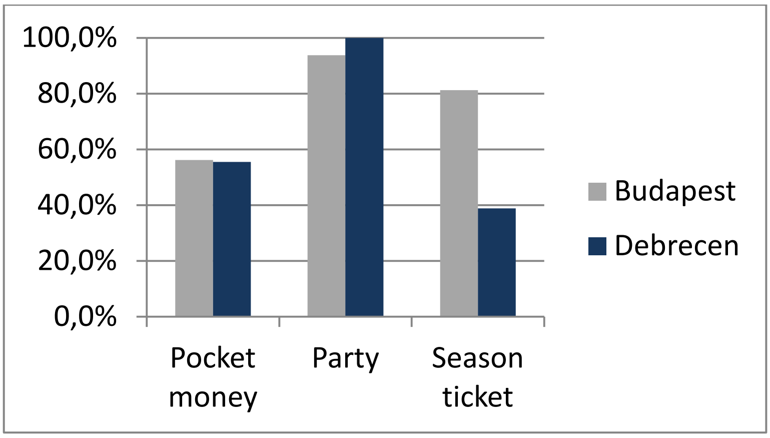


Figure 5: The open answers coding by X1 or 3X

Near to the 50% of the students recognised the openness of the „Pocket money" problem in both groups. The simple „Party" problem seemed to be open for near to the 100% of the students. The more complex „Season ticket" problem was detected as open in more cases in Budapest (BP-group) as in Debrecen (DE-group) Probably the BP-group was more familiar with this real-world situation as it turned out from the commentar of a teacher student in DE group:

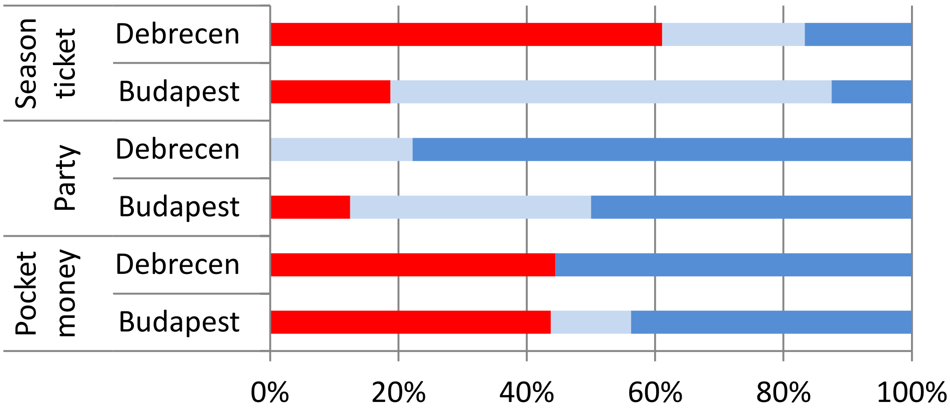
„In my opinion this is an open task. I usually buy such a ticket for the train, …, it may happen that I haven't ticket even for two weeks. …"

We classified the answers into three groups concerning their code in the following way:

- Closed answer means that the codes are 10, 20, 40.
- Correct open answer or at least partly correct open answer means that the codes are 30 or 31.
- The answer reflects for hesitation concerning the closed answer means that the codes are 11, 21, 41.

We detected similarities and differences between the achievements of the two groups (Figure 7):

- The pattern of the results of the „Pocket money" problem is similar.
- For the „Party" problem in the BP-group there exists closed answer, while in the DE-group not.
- For the „Season ticket" problem about 60% of the member of the DE-group gave closed answer. In the BP-group there are more students who formulated only hesitation and didn't give (partly) correct answer than in the DE-group.



closed: ; hesitation: ; correct (or partly correct) open

Figure 7: From the closed towards the open solution.

We also investigated the effect of our teaching concerning the recognising of the openness of the problems (realistic answers). Table 3 give some overview about the number of students whose answers fit into one of the following five categories: (1) Open-Open-Open (OOO); (2) Open-Open-Closed (OOC); (3) Open-Closed-Open (OCO); (4) Closed-Open-Open (COO); (5) Closed-Open-Closed (COC).

Problem	Pocket money	Open	Open	Open	Closed	Closed
	Party	Open	Open	Closed	Open	Open
	Season ticket	Open	Closed	Open	Open	Closed
Group	Budapest	7	1	1	5	1
	Debrecen	4	6	0	3	5

Table 2: The “effect” of our teaching.

We were very happy of course that no one was in the category CCC, so everybody gave at least once open answer. We can detect development regarding the recognition of the openness of a problem in the work of those students whose answers were in the categories „COO” and „COC”. The fact that 6 students from Debrecen were in the category „OOC” probably means that they were not familiar with the “Season ticket” situation in their everyday life or even the closed version of the task was too complicated for them and they got lost in the details.

At the end of the semester we discussed with the students what they are thinking about open problems and their role in the process of learning mathematics. Here we quote some of the students’ opinions:

- „The tasks are interesting and useful.”

- [Tasks like these] „develop thinking, so they are useful."
- [Tasks like these] „help us to formulate tasks without misunderstanding."
- [Tasks like these] „help us not to evaluate a situation rashly 'too easy'."
- „Solving tasks like these, where there are several possibilities for a correct solution, students probably don't worry if they couldn't find the only one."
- „The tasks are interesting but I don't know what they are proper for."
- „I hate this kind of tasks because there is no mathematics in them!"

As we expected, the opinions cover a wide range. Some remarks are in close connection with the concrete tasks, while others refer to the belief of the teacher students about mathematics (for example the last one). Furthermore the answers show that the viewpoint of the students is different; some of them evaluated the problems from a student's perspective, while others from the perspective of a future teacher.

CONCLUSION

The experience with the tasks showed our students that similarly to younger pupils they think schematically time to time. It also happened that they got lost in the details or overemphasized the real situations. The tasks then the discussions helped students to recognize whether a problem is open or not. If they are familiar with a situation in their everyday life, they recognize it easier in the math problem. We agree with Cheng (2013), that „Students solutions of problems embedded in real life context often reflected their personal values and beliefs." (Chan, 2005 quoted by Cheng, 2013)

The simple problems (with simple calculations) may indicate the idea of openness, because otherwise it is too easy for them. From this experience arises the question we have to investigate in the future: What about the open problems with more mathematics?

Sometimes the teacher students hesitated even if they gave open answer, eg. „If I think that this is an open problem [the „Season ticket"], shall I also write the close solution just in case?" They are not quite sure that this kind of solution is acceptable, they are afraid that the teacher thinks on the closed version. Here we point out that solving problems is based how one thinks about mathematics. The phenomenon refers to that belief system which often appears concerning in-service teachers too. We determine as an important task to change this rigid belief system concerning the open reality based problems and to teach our future teachers how to use their mathematical knowledge even in everyday (real) situations. We emphasize that teacher students have to be familiar with the real situation based problems during the training already in order to integrate it into their prospective teacher's belief system.

As a final remark we have to mention that five sessions in a semester are enough to develop awareness of future teachers towards open real-life problems, but not enough to change their belief and behaviour concerning the math problem solving routine. This is the reason we plan further developmental program with different problems and structure in order to better understand students' thinking and the way of the effective contribution to give our students wider perspective in this topic.

Acknowledgement

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