Final exam questions

Applied Mathematics MSc

Students draw a random question from the entire list, and after a certain preparation period, give an account on it. The committee gives a single mark for the student's answers in the final exam.

1. **Graph theory and applications**
Graph colourings, chromatic number, Mycielski construction, perfect graphs, chromatic polynomial, chromatic index. Independence and covering. Perfect matchings in bipartite and in arbitrary graphs.

2. **Algorithms in mathematics**
The sorting problem. Insertion-sort, Merge-sort, Heapsort, Quicksort. Lower bound for the running time of sorting. Sortin gin linear time. Data structures (stacks, queues, linked lists, rooted binary trees, rooted trees with unbounded branching).

3. **Convex optimization**
Separation theorems for convex sets and convex functions, corollaries. Directional derivative and subgradient of convex functions, rules of calculus. Convex programming tasks, Kuhn-Tucker-theorem.

4. **Discrete optimization**
Assignment problem, set covering problem, Chinese postman problem, travelling salesman problem. Max flow−min cut problem, Ford-Fulkerson theorem, Edmonds-Karp theorem.

5. **Applications of ordinary differential equations**
Autonomous differential equations (autonomous equations, equilibrium states, limit cycles and phase spaces). Stability of ordinary differential equations (Theorems of Lyapunov, Lyapunov's direct method). Boundary value problems and eigenvalue problems (boundary value problems, Sturm boundary value problems, fundamental solutions, Green functions, non-linear boundary value problems, maximum and minimum principles, Sturm--Liouville eigenvalue problems). Calculus of variations (variation of a functional, bi-linear and quadratic functionals, second order variation of a functional, extrema of functionals, Euler--Lagrange differential equations)

6. **Partial differential equations**
Classification and canonical form of linear and quasi-linear second order partial differential equations. Problems for elliptic, parabolic and hyperbolic equations (initial value problems, boundary value problems, maximum theorem).

7. **Stochastic processes**
Gaussian processes, Wiener processes and main properties. Martingales, martingale inequalities and convergence theorems.

8. **Multivariate analysis**
Principal component analysis, factor analysis, classification methods (maximum likelihood method and Bayesian decision), cluster analysis (hierarchical and K-means methods).

9. **Derivative securities**
Derivative assets, futures, forward and option contracts. Pricing futures and forward contracts, basic properties of option prices (effects of variables on price, early exercise, bounds, put-call parity). Monte Carlo simulation for option pricing.

10. **Discrete time markets**
Discrete time markets, binary and binomial markets, trading strategies. Equivalent martingale measures, arbitrage and completeness of the market and the related fundamental theorems of asset pricing, main theorems of option pricing in discrete markets.

11. **The Black-Scholes market**
Stock price models in continuous time, Ito processes, Ito formula. The Black-Scholes model, volatility and its estimations, the Black-Scholes equality and the Black-Scholes pricing formula.

12. **Econometrics**
The standard linear model, assumptions, hypothesis tests in the standard linear model. Testing the stability of parameter estimates. Heteroscedasticity, tests and methods: WLS, GLS and FGLS estimations. Examples.

13. **Game theory**
Normal form of non-cooperative games. The Nash equilibrium. Analysis of finite games. Application of the game theoretic approach to simpler market modells. Two player zero sum games, matrix games. Cooperative games.